EXAMPLES IN THEORY OF MACHINES PROBLEMS

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PREFACE

THESE examples in Theory of Machines problems have been collected and classified for the guidance of students preparing for examinations for Higher National Certificates, Associate-Membership of the Institutions of Mechanical and Civil Engineers, and degrees in engineering.

Many examples have been selected from past examination papers of the Institutions of Mechanical and Civil Engineers, London University, the Mechanical Science Tripos, Cambridge, and the examinations for Whitworth and Senior Whitworth Scholarships. Acknowledgments are made to the authorities of these Institutions, London University, Cambridge University and to the Controller of H.M. Stationery Office, for permission to reproduce the questions. Thanks are also due to Messrs. Wm. Clowes & Sons, Ltd., for authority to reprint the questions from past examination papers of the Institution of Civil Engineers.

A large number of worked examples have been included with a view to assisting those students who are engaged in private study, and finally it is hoped that errors, which are difficult to eliminate, will be few.

W. R. C.

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SYMBOLS AND ABBREVIATIONS USED IN THE TEXT

Symbols

s, x, y . . .	distance.
t	time.
u, v, V . .	velocity.
f	acceleration.
g . .	acceleration due to gravity.
$\alpha, \beta, \gamma, \theta, \phi$ . .	angle.
ω	angular velocity.
$\dot{\omega}$	angular acceleration.
M, m . . .	mass.
W, w . . .	weight.
F	force.
μ	coefficient of friction.
$\bar{x}, \bar{y}$	distance to centre of gravity.
I	moment of inertia.
k	radius of gyration.
ρ	mass of unit volume.
Σ	summation sign.
η	efficiency.
N	revolutions per minute.
n	revolutions per second.
l	length.
R, r . . .	radius.
C	modulus of rigidity.
E	Young's modulus.
T	tension.

Abbreviations

in.	inches.
ft.	feet.
yd.	yards.
ml.	miles.
sec.	seconds.
min. . . .	minutes.
ft./sec. . . .	feet per second.
m.p.h. . . .	miles per hour.
ft./sec ^{2} . . .	feet per second per second.
r.p.m. . . .	revolutions per minute.
radn. . . .	radians.
h.p. . . .	horse-power.
c.g. . . .	centre of gravity.
k.e. . . .	kinetic energy.
p.e. . . .	potential energy.
lb.	pounds.
L.U. . . .	London University.
L.U.A. . . .	London University Advanced.
W.S. . . .	Whitworth Scholarship.
W.S.S. . . .	Whitworth Senior Scholarship.
I.M.E. . . .	Institution of Mechanical Engineers.
I.C.E. . . .	Institution of Civil Engineers.
Mech. Sc. Tripos Cam.	Mechanical Science Tripos Cambridge.

EXAMPLES IN THEORY OF MACHINES PROBLEMS

1. MOTION—VECTORS—FORCE—MOMENTUM

1. A motor car starts from rest on a straight horizontal road with a constant acceleration in bottom gear and reaches 15 m.p.h. in 30 sec. The driver then changes gear and reaches 45 m.p.h. in a further 45 sec. Find the total distance covered and the constant accelerations in bottom and second gears respectively.

SOLUTION. For motion in a straight line with constant acceleration we have the relations,

$$v = u + ft \qquad (a)$$

$$s = ut + \tfrac{1}{2}ft^2 \qquad (b)$$

where v and u are the final and initial velocities respectively, s the distance covered in time t, and f the acceleration.

(i) Bottom gear.

$$v_0 = \text{final velocity} = \frac{88}{60} \times 15 = 22 \text{ ft./sec.}$$

$$u_0 = \text{initial velocity} = 0 \text{ since car starts from rest.}$$

$$t_0 = \text{time to attain } v_0 = 30 \text{ sec.}$$

Hence substituting in (a) and (b) we have,

$$22 = 0 + 30f_0 \quad \therefore\ f_0 = \frac{22}{30} = \frac{11}{15} \text{ ft./sec.}^2$$

$$s_0 = 0 + \frac{1}{2}\left(\frac{11}{15} \times 900\right) = 330 \text{ ft.}$$

(ii) Second gear.

$$v_1 = \text{final velocity} = \frac{88}{60} \times 45 = 66 \text{ ft./sec.}$$

$$u_1 = \text{initial velocity} = 22 \text{ ft./sec.}$$

$$t_1 = \text{time to attain } v_1 = 45 \text{ sec.}$$

Again substituting in (*a*) and (*b*) we have,

$$66 = 22 + 45f_1 \quad \therefore f_1 = \frac{44}{45}\text{ft./sec.}^2$$

$$s_1 = (22 \times 45) + \frac{1}{2}\left(\frac{44}{45} \times 45 \times 45\right) = 1\,980 \text{ ft.}$$

The total distance covered $= s_0 + s_1 = (330 + 1\,980) = 2\,310$ ft.

The accelerations required are $f_0 = \frac{11}{15}$ ft./sec.^{2} and $f_1 = \frac{44}{45}$ ft./sec.^{2}

2. Two railway stations are 3 miles apart; a train starts from one with a constant acceleration of 0·5 ft./sec.^{2} which is maintained for 1 mile, after which the train proceeds at constant speed for 0·75 mile. The brakes are then applied and the train slows down with constant retardation to come to rest at the second station.

Find the total time for the journey, the maximum speed, and the retardation during the last part of the journey.

Ans. 5·732 min.
49·56 m.p.h.
0·4 ft./sec.^{2}

3. The distance moved by a motor car in time *t* seconds from rest on the horizontal is given by the series of values of *s* and *t* below.

Plot speed-time, and acceleration-time curves for the motion, and find the velocity and acceleration at 0·75 sec. from rest.

SOLUTION. An approximate solution of this problem may be obtained by taking first and second differences of the displacement *s*. Arrange in tabular form as below.

Time t sec.	Distance s ft.	δt	δs	$v = \dfrac{\delta s}{\delta t}$	δv	$f = \dfrac{\delta v}{\delta t}$
0	0					
		0·10	0·40	4·0		
0·1	0·40				1·5	15
		0·10	0·55	5·5		
0·2	0·95				2·0	20
		0·10	0·75	7·5		
0·3	1·70				− 0·5	− 5
		0·10	0·70	7·0		
0·4	2·40				− 2·0	− 20
		0·10	0·50	5·0		
0·5	2·90				− 0·5	− 5
		0·10	0·45	4·5		
0·6	3·35				− 2·0	− 20
		0·10	0·25	2·5		
0·7	3·60				− 0·3	− 3
		0·10	0·22	2·2		
0·8	3·82				− 1·0	− 10
		0·10	0·12	1·2		
0·9	3·94				− 0·1	− 1
		0·10	0·11	1·1		
1·0	4·05					

The differences of t are given by $\delta t = (t_2 - t_1)$, $(t_3 - t_2)$, etc. The corresponding first differences of s are $(s_2 - s_1)$, $(s_3 - s_2)$, etc. The speeds, $v = \delta s/\delta t$, are the mean speeds during the interval δt and must be plotted on the middle ordinate of this time interval.

The second differences δv are given by $(v_2 - v_1)$, $(v_3 - v_2)$, and so on. These denote the increase in speed from the middle of one time interval to the middle of the next. The mean acceleration during this period is $\delta v/\delta t$ and the first value of this must be set up on the second time ordinate and so on.

The curves should be plotted and the velocity and acceleration, at time 0·75 sec. from rest, read off.

4. The distance moved by a particle in time t sec. from rest along a straight line is given by

$$s = a \sin pt.$$

Find expressions for the velocity and acceleration at time t and show that the acceleration is always directed towards a fixed point and proportional to the distance to the particle from this point.

5. A train is moving along a straight horizontal track at 60 m.p.h. The brakes are applied and the retardation after time t is given by $f = 6t$. Find the time required to stop the train and the distance travelled in this time.

Hint: Note that retardation $= -(d^2s/dt^2) = 6t$.

Ans. $t = 5{\cdot}42$ sec.
$s = 317{\cdot}8$ ft.

6. A train shuts off steam and is uniformly retarded. In 40 sec. it covers 0·5 mile, and in another 30 sec. it comes to rest. Draw the velocity-time graph, and determine, (i) the velocity in ml./min. units of the train when steam was shut off, (ii) the retardation in ml./min.^{2} units and in ft./sec.^{2} units, and (iii) the total distance covered by the train after the steam was shut off. (*W.S.*)

Ans. (i) 1·05 ml./min.
(ii) 0·015 ml./min.^{2} and 1·32 ft./sec.^{2}
(iii) 3 234 ft.

7. The distance between successive floors of a building is 12 ft. A lift starts upwards from the ground floor with a uniform acceleration of 16 ft./sec.^{2}, and is then subjected to a retardation of equal amount which brings it to rest at the first floor. Draw a speed-time graph for the motion, and find the average speed of the lift. (*W.S.*)

SOLUTION. Obviously the accelerating period is equal to the retarding period, i.e. the acceleration continues for a distance of 6 ft. after which the lift is retarded for the remaining 6 ft.

Making use of the relation

$s = \frac{1}{2}ft^2$ for motion under constant acceleration from rest

we have $\quad 6 = \frac{1}{2} \times 16 \times t^2.$

$\therefore \quad t^2 = 6/8$ and $t = (\sqrt{3})/2$ sec.

The total time between the two floors is therefore

$$2t = \sqrt{3} = 1{\cdot}732 \text{ sec.}$$

From $t = 0$ to $t = 0{\cdot}866$ sec. the velocity of the lift is given by

$$v = ft = 16t \text{ ft./sec.}$$

From $t = 0{\cdot}866$ to $t = 1{\cdot}732$ sec. the velocity is given by

$$v = 13{\cdot}856 - 16(t - 0{\cdot}866)$$
$$= 27{\cdot}712 - 16t \text{ ft./sec.}$$

The velocity-time graph may now be drawn. It will be seen that this consists of two straight lines. The average speed is thus half the maximum

$$= 13{\cdot}856/2 = 6{\cdot}928 \text{ ft./sec.}$$

8. An aeroplane is approaching a railway line at right angles and 500 ft. above the ground. The engine of a train will be directly under the plane as it passes over the line. If the speed of the plane is 120 m.p.h., at what instant must the pilot release a bomb designed to strike the engine? At what distance is he then from the engine if the speed of the latter is 30 m.p.h.?

SOLUTION. This is a case (Fig. 1) of motion under gravity with an initial horizontal velocity. Considering the horizontal motion, it is obvious that the horizontal velocity will remain constant throughout the motion.

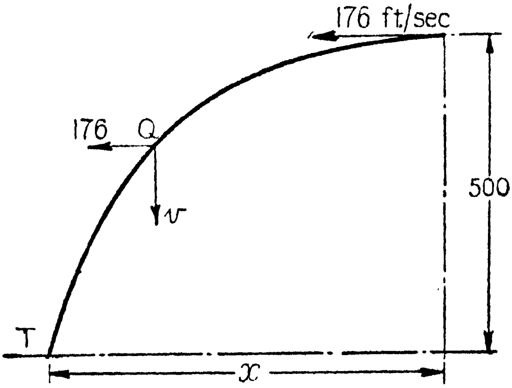

FIG. 1

The vertical motion is controlled by gravity, i.e. the acceleration is constant and $= g$, hence to get the time of descent we have,

$$s = ut + \tfrac{1}{2} gt^2$$
$$\text{or } 500 = 0 + \tfrac{1}{2} \times 32{\cdot}2 \times t^2$$

since initial vertical velocity $= 0$.

$$\therefore \qquad t = \sqrt{(1\,000/32{\cdot}2)} = 5{\cdot}57 \text{ sec.}$$

before crossing line.

The horizontal distance from the line at this instant is given by

$$x = 176 \times 5{\cdot}57 = 980{\cdot}6 \text{ ft.}$$

The engine is a distance

$$y = 44 \times 5{\cdot}57 = 245{\cdot}15 \text{ ft.}$$

from the point of impact.

Hence the distance from the plane to the engine is

$$d = \sqrt{[x^2 + y^2 + (500)^2]}$$

$$= \sqrt{[(980{\cdot}6)^2 + (245{\cdot}15)^2 + (500)^2]}$$

$$\therefore \quad d = 1\,128 \text{ ft.}$$

9. A flywheel rotating at 1 000 r.p.m. is brought to rest in 20 sec. by a uniform angular retardation. Find the total number of revolutions made during this time and the value of the retardation.

Ans. 166·6 rev.
5·23 radn./sec.^{2}

10. An aeroplane flies in a straight line from East to West a distance of 100 ml. in 1 hr. If there is a following wind blowing from the North-East at 30 m.p.h., in what direction must the pilot steer, and at what speed?

SOLUTION. This is a case of vector addition, thus

$$\overrightarrow{\text{Velocity of plane relative to earth}}$$

$$= \begin{cases} \overrightarrow{\text{Velocity of plane relative to air}} + \\ \overrightarrow{\text{Velocity of air relative to earth}} \end{cases}$$

or $$\vec{V}_e = \vec{V}_p + \vec{V}_a$$

$$V_e = 100 \text{ m.p.h. from East to West.}$$

$$V_a = 30 \text{ m.p.h. from North-East.}$$

A semi-graphical method (Fig. 2) is easiest to follow.

Resolving horizontally,

$$30 \cos 45^\circ + V_p \cos \theta = 100$$

$$30/\sqrt{2} + V_p \cos \theta = 100$$

$$\therefore \qquad V_p \cos \theta = 78{\cdot}79 \qquad (a)$$

Resolving vertically,

$$30 \sin 45^\circ = V_p \sin \theta$$

$$\therefore \qquad V_p \sin \theta = 21{\cdot}21 \qquad (b)$$

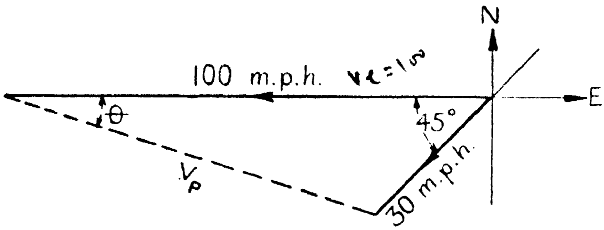

FIG. 2

Dividing (b) by (a),

$$\tan \theta = \frac{21{\cdot}21}{78{\cdot}79} = 0{\cdot}2692$$

$$\therefore \qquad \theta = 15{\cdot}1^\circ$$

Hence $\quad V_p = \dfrac{78{\cdot}79}{\cos 15{\cdot}1^\circ} = \dfrac{78{\cdot}79}{0{\cdot}9655} = 82{\cdot}52$ m.p.h.

The pilot must steer $15{\cdot}1^\circ$ North of West at $82{\cdot}52$ m.p.h.

11. An automatic machine is tapping $\frac{1}{2}$ in. nuts. The tap and the nut rotate in the same direction, and the nut runs at a constant speed of 100 r.p.m. The tap cuts 10 threads in 10 sec., its speed is then raised and it is withdrawn in 4 sec. Find the speeds of the tap.

SOLUTION. This is a case of relative angular motion. Let the nut rotate anti-clockwise.

(i) N_1 = speed of tap when cutting, also anti-clockwise. Then speed of nut relative to tap = $(100 - N_1)$ r.p.m. anti-clockwise.

The nut advances 10 threads in 10 sec. = 1 rev./sec., hence

$$(100 - N_1) = (10/10) \times 60$$

$$100 - N_1 = 60$$

$$\therefore \qquad N_1 = 40 \text{ r.p.m.}$$

(ii) N_2 = speed of tap when withdrawing, anti-clockwise. Then speed of tap relative to nut = $(N_2 - 100)$, the tendency being to unscrew.

The tap unscrews 10 threads in 4 sec., hence

$$(N_2 - 100) = (10/4 \times 60)$$

$$N_2 - 100 = 150$$

$$\therefore \qquad N_2 = 250 \text{ r.p.m.}$$

12. A plane flying over a straight horizontal course of length L was timed with and against a steady wind blowing along the course. The times taken were t_1 and t_2 respectively. Show that the speed V_p of the plane relative to the air is given by

$$V_p = L(t_1 + t_2)/2t_1t_2$$

and that the total time for the double journey in still air was less than $(t_1 + t_2)$.

(*Hint:* Show that $(t_1 - t_2) > 0$ for the last part of problem.)

13. A small drop of rain striking the window of a railway carriage marks out a short straight line on the glass. It is noticed that lines formed when the train is at rest are at right angles to those formed when the train is at full speed, pairs of lines forming crosses with arms at 45° to the horizontal. Find the vertical speed of the rain if the train travels at a full speed of 40 m.p.h. (*W.S.*)

SOLUTION. Vectorially,

$$\overrightarrow{\text{Rain relative to train}} + \overrightarrow{\text{Train relative to ground}}$$

$$= \overrightarrow{\text{Rain relative to ground}}.$$

Again using a semi-graphical method we have

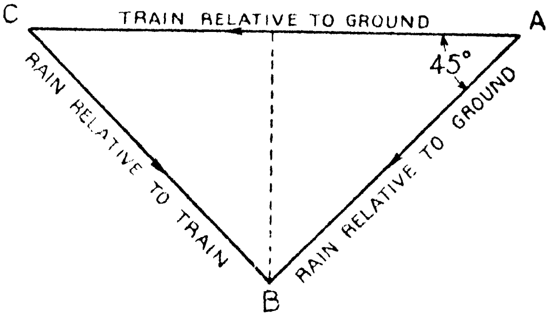

FIG. 3

The vertical velocity of the rain (Fig. 3) is obviously AB sin 45°.

$$AB \cos 45° = AC/2 = 40/2 = 20 \text{ m.p.h.}$$

$$\therefore \qquad AB = 20\sqrt{2} \text{ m.p.h.}$$

and so $\qquad AB \sin 45° = (20\sqrt{2}) \times (1/\sqrt{2}) = 20 \text{ m.p.h.}$

14. Sketch a graphical method of calculating the angle of the cross in the previous problem where the train is travelling at any given speed and evaluate for $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ full speed. (*W.S.*)

Ans. Vertical angle of cross $= (\pi/4) + \tan^{-1}(v/20 - 1)$.
16·4°; 45°; 73·6°.

15. A lift cage weighing 2 tons is accelerated upwards by means of a vertical rope from rest to 30 ft./sec. in 4 sec. with constant acceleration. Find the tension in the rope during this period, neglecting the weight of the rope.

SOLUTION. We require first of all (Fig. 4) the acceleration of the cage. Thus

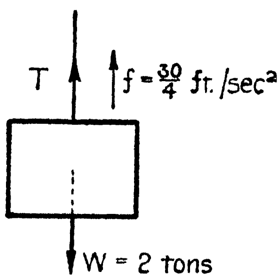

FIG. 4

$$v = u + ft$$

$$30 = 0 + f \times 4$$

$$\therefore \qquad f = 30/4 \text{ ft./sec.}^2$$

Making use of the relation

$$\text{Force} = \text{Mass} \times \text{Acceleration}.$$

$$(T - W) = (W/g) \times f \text{ where } g \text{ is the acceleration due to gravity.}$$

$$(T - 2) = (2/32 \cdot 2) \times 30/4 = 15/32 \cdot 2$$

$$\therefore \quad T = (2 + 15/32 \cdot 2) = 2 \cdot 47 \text{ tons.}$$

16. A heavy ball of weight 120 lb. is suspended from a fixed point by a string of length 3 ft., and is rotating about a vertical axis through this point with a uniform angular velocity $\omega = 10$ radn./sec. Find the angle between the cord and the vertical axis, and the tension in the cord if its weight may be neglected.

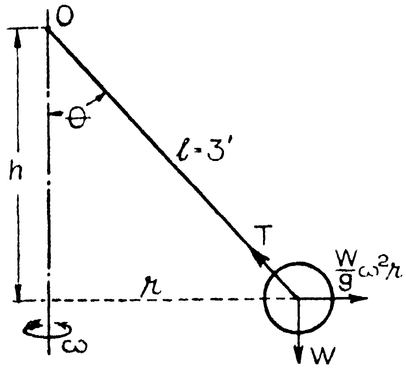

FIG. 5

SOLUTION. Since the ball describes a circle it has a radial acceleration $\omega^2 r$ directed towards the axis of rotation, which gives rise to the fictitious centrifugal force $(W/g)\omega^2 r$ acting radially outwards on the ball.

The forces acting on the ball are as shown (Fig. 5). Taking moments about the fixed point O, we have

$$(W/g) \cdot \omega^2 \cdot r \times h = W \cdot r$$

$$\omega^2 \cdot h/g = 1 \text{ and } h = l \cos \theta$$

$$\therefore \quad l \cos \theta = g/\omega^2.$$

Substituting the given values,

$$3 \times \cos \theta = 32 \cdot 2/(10)^2$$

$$\cos \theta = 32 \cdot 2/300 = 0 \cdot 1073$$

$$\therefore \quad \theta = 83 \cdot 8°.$$

If T is the tension in the string we have, resolving vertically forces on the ball,

$$T \cos \theta = W$$

$$\therefore \quad T = W/\cos \theta = 120/0 \cdot 1073 = 1\,119 \text{ lb.}$$

17. For a certain type of locomotive and tender the total resistance R lb. per ton at V m.p.h. is given by

$$R = 8\cdot8 + 0\cdot01V + 0\cdot004V^2.$$

Find approximately by Simpson's rule, or otherwise, the distance covered on the level after steam is shut off, during a reduction of speed from 70 m.p.h. to 30 m.p.h. (*W.S.*)

Ans. 1·82 ml.

18. For a certain motor car, the road speed (v ft./sec.) and the corresponding brake horse-powers on top gear and full throttle, are given in the following table.

v ft./sec.	20	30	40	50	60
B.H.P.	7·1	11·7	15·6	18·8	21·3
F lb.	86	104	122	140	158

For a loaded car weighing 1 ton and under certain road and wind conditions, the total resistance to motion, estimated as a tractive resistance F lb., is also given. Calculate and enter up the accelerations of the car at the several speeds and exhibit the result in a graph plotted on a speed base.

On the same base show a time curve; read off the time spent in increasing the speed from 20 to 60 ft./sec. (*W.S.*)

19. A vessel of 800 tons displacement gradually comes to rest against the resistance of the water, which may be taken to vary as the speed.

Measuring from a certain instant the distance covered in 30 sec. is 100 ft., and the distance in 1 min. is 140 ft.

Determine the resistance at 1 ft./sec. and the speed at the beginning of the measured intervals. (*W.S.*)

SOLUTION. We make use of the relation,

$$\text{Resistance} = \text{Mass} \times \text{Retardation}$$

remembering that the resistance varies with the speed.

Thus, resistance $= k \times$ speed $= k \, . \, (ds/dt)$,

retardation $= -\,(d^2s/dt^2)$,

$$\text{mass} = \frac{(800 \times 2\,240)}{32\cdot2} = 55\,640 \text{ lb.}$$

Hence the equation of motion is,

$$k \,.\, (ds/dt) = -\,55\,640 \,.\, (d^2s/dt^2)$$

$$\text{or} \quad \frac{d^2s}{dt^2} + \frac{k}{55\,640} \cdot \frac{ds}{dt} = 0.$$

The solution of this equation is

$s = A + Be^{-(kt/55\,640)}$ where A and B are constants.

At the beginning of the measured intervals $t = 0$ and $s = 0$ which leads to

$$B = -\,A, \text{ so that}$$

$$S = A[1 - e^{-(kt/55\,640)}]. \qquad \text{(i)}$$

When $t = 30$ sec., $s = 100$ ft.
When $t = 60$ sec., $s = 140$ ft., and substituting these values in (i) we get

$$100 = A[1 - e^{-(30k/55\,640)}]$$

$$140 = A[1 - e^{-(60k/55\,640)}].$$

By division $\quad 1{\cdot}4 = \dfrac{[1 - e^{-(30k/55\,640)}]}{[1 - e^{-(60k/55\,640)}]}$ which after some reduction leads to

$$k = 1\,699 \text{ lb./(ft./sec.)} = \text{resistance at 1 ft./sec.}$$

Speed at beginning of measured intervals is thus

$$ds/dt \text{ at } t = 0, \text{ which is}$$

$$Ak/55\,640 = 5{\cdot}088 \text{ ft./sec.}$$

20. It is found that an unbalanced force of 3 lb. causes an acceleration of 150 ft./sec.^{2} of a certain particle. If this particle moves at a constant speed of 120 ft./sec. along a circular path of 4 ft. diameter, find the unbalanced force necessary to produce this motion. Illustrate with a diagram. (*W.S.*)

SOLUTION. (Fig. 6.) The first part of the data enables us to find the mass of the particle.

Since $\qquad$ Force = Mass × Acceleration we have,

$$3 = M \times 150$$

$$\therefore \quad M = 3/150 = 1/50 \text{ lb.}$$

The unbalanced force necessary to produce the circular motion is the centripetal force on the particle.

$$F = M\omega^2 . r$$

where ω = angular velocity of the particle.

Now $v = \omega . r$

$\therefore \quad 120 = \omega \times 2$

$\therefore \quad \omega = 60$ rad./sec.

and $F = (1/50) . (60)^2 . 2$

$= 3\,600/25$

$= 144$ lb. wt.

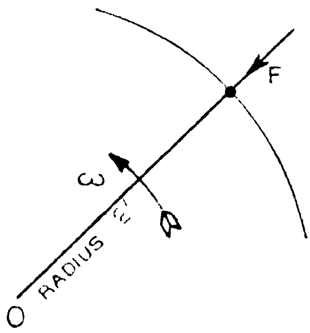

FIG. 6

21. A body moves at constant speed in a circular path of radius 5 ft. in a horizontal plane. In an interval of 0·2 sec. the radius sweeps out an angle of 10°. Find the mean acceleration of the body and state the magnitude, direction, and sense of the force required to produce this acceleration in a body weighing 2 lb.

Ans. 3·805 ft./sec.^{2}
0·236 lb.

22. The valve of a petrol engine is closed by means of a spring which has a stiffness of 30 lb./in. compression. If the valve opens downwards and its full lift is $\frac{1}{4}$ in. and its weight 0·3 lb., find the time taken by the spring to close the valve. When closed the compression is $\frac{1}{2}$ in.

SOLUTION. Let x ft. be the displacement of the valve from full lift position at time t sec. When closed $x = \frac{1}{48}$ ft. In any position the closing force on the valve is

$$F = 30\left(\tfrac{1}{2} + \tfrac{1}{4} - 12x\right) - 0{\cdot}3 \text{ lb.}$$
$$= 22{\cdot}2 - 360x \text{ lb.}$$

The acceleration of the valve is d^2x/dt^2 and since

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$
$$22{\cdot}2 - 360x = (0{\cdot}3/32{\cdot}2)(d^2x/dt^2)$$

or rewriting the equation of motion,

$$d^2x/dt^2 + 38\,640x = 2\,383$$

The solution of this differential equation is,

$$x = \mathrm{A} \cos (\sqrt{38\,640}) \,.\, t + \mathrm{B} \sin (\sqrt{38\,640}) \,.\, t + \frac{2\,383}{38\,640}$$

where A and B are constants of integration.

$$\text{At } x = 0,\ t = 0. \quad \therefore\ \mathrm{A} + \frac{2\,383}{38\,640} = 0, \text{ or } \mathrm{A} = -\frac{2\,383}{38\,640}.$$

At $t = 0$, $dx/dt = 0$ since valve starts from rest and so $\mathrm{B} = 0$, and the complete solution is,

$$x = \frac{2\,383}{38\,640}\,[1 - \cos(\sqrt{38\,640}) \,.\, t].$$

The time of closing is obtained from the relation

$$t = t_1 \text{ when } x = 1/48 \text{ ft.}$$

$$\therefore \qquad \frac{1}{48} = \frac{2\,383}{38\,640}\,[1 - \cos(\sqrt{38\,640}) \,.\, t_1]$$

$$\cos(\sqrt{38\,640}) \,.\, t_1 = 0{\cdot}6622$$

$$(\sqrt{38\,640}) \,.\, t_1 = 0{\cdot}8465$$

$$\therefore \qquad t_1 = 0{\cdot}0043 \text{ sec.}$$

23. A valve is opened by a cam and closed by a helical spring. The weight of the valve is 1 lb.; the initial force on the spring when the valve is closed is 6 lb.; the stiffness of the spring is 10 lb./in. and the spring has 20 convolutions. The constant frictional resistance to the motion of the valve is 0·75 lb.

(*a*) Find the time taken to close the valve from the position of maximum lift, which is 0·25 in., and

(*b*) find the diameter of the steel wire of which the spring is made. (*L.U.A.*)

Ans. (*a*) 0·0126 sec.
(*b*) $\frac{1}{16}$ in.

(*Hint*. As in Problem 22 but with resistance.)

24. A train weighing 100 tons is ascending an incline of 1 in 120. The engine exerts a constant tractive force of 1·5 tons wt. and the resistance due to friction is 10 lb. wt.

per ton. Find the acceleration of the train along the incline.

Ans. 0·071 ft./sec.^{2}

25. A wagon weighing 10 tons is shunted on to an incline of 1 in 80 at a speed of 15 m.p.h., and comes to rest in a distance of 100 yd. Find the resistance of the rails to motion (assumed constant) and the time taken to come to rest.

Ans. 28 lb. wt. per ton.
27·3 sec.

26. A train of 50 tons weight moving at 10 m.p.h. strikes a buffer stop and is brought to rest in a distance of 2 ft. Assuming the force exerted to be constant, find the force and the time taken to bring the train to rest. (*I.M.E.*)

SOLUTION. Let F be the required force, then since

$$\text{Force} = \text{Mass} \times \text{Retardation}$$

$$F = \frac{50 \times 2\,240}{32 \cdot 2} \cdot f = 3\,477\,f.$$

Now $v^2 = u^2 - 2fs$ and $u = 10$ m.p.h. $= 14 \cdot 66$ ft./sec.

$$0 = (14 \cdot 66)^2 - 2 \times 2 \times f$$

$$\therefore \quad f = (14 \cdot 66)^2/4 = 53 \cdot 71 \text{ ft./sec.}^2$$

Hence $$F = \frac{3\,477 \times 53 \cdot 71}{2\,240} = 83 \cdot 41 \text{ ton wt.}$$

The time taken is given by

$$v = u - ft$$

$$0 = 14 \cdot 66 - 53 \cdot 71t$$

$$\therefore \quad t = 14 \cdot 66/53 \cdot 71 = 0 \cdot 2729 \text{ sec.}$$

27. A train weighs 400 tons, and the locomotive and tender 110 tons. The maximum tractive effort is a sixth of the weight on the driving wheels. Find the minimum weight on the driving wheels so that the locomotive and train can get up a speed of 60 m.p.h. from rest in 3 min., assuming the wind resistance to be 8 lb. wt. per ton and an incline of 1 in 500. (*I.M.E.*)

Ans. 63·5 tons.

28. It is suggested that emergency buffers might be designed so that the impact of a train would be absorbed in extending two long bars of ductile mild steel, one bar being used in each of the two buffers. If the bars are 10 ft. long and 2 in. diameter, and if the steel is such that it elongates 25% with a mean stress of 24 tons/sq. in. before fracturing, find the highest speed from which a 400 ton train can be stopped before the bars break. *(I.C.E.)*

Ans. 3·756 m.p.h.

29. Two stations on a railway are $\frac{1}{2}$ mile apart. The maximum speed of a train between the stations is 40 m.p.h. The accelerating force on the train is $\frac{1}{8}$, and the braking force $\frac{1}{4}$, of the weight of the train. Find the minimum time to travel from station to station. *(I.M.E.)*

Ans. 55·91 sec.

30. A jet of steam flowing from a nozzle at the rate of 0·473 lb./sec. is directed normally against a flat plate and leaves the latter tangentially. The recorded force acting on the plate being 16·1 lb., estimate the speed of the jet. *(W.S.)*

Solution. The recorded force on the plate is equal to the rate of change of linear momentum of the jet, resolved normal to the plate.

Since the jet leaves the plate tangentially, the final momentum has no component normal to the plate.

Hence,

$$\text{Force} = \text{Change of Momentum/second (normal to plate)}$$
$$= (\text{Mass/second}) \times \text{Initial velocity}$$

$$\therefore \quad 16{\cdot}1 = (0{\cdot}473/32{\cdot}2) \times v$$

and $$v = \frac{16{\cdot}1 \times 32{\cdot}2}{0{\cdot}473} = 1\,096 \text{ ft./sec.}$$

31. A jet of steam flowing from a nozzle at the rate of 0·5 lb./sec. and a velocity of 1 000 ft./sec. strikes a curved plate and passes over it without loss of speed but with a change of direction of 45.° Find the resultant force on the plate.

SOLUTION. The resultant force is equal to the rate of change of momentum of the jet. Since momentum is a vector quantity, this change must be found vectorially.

From the vector diagram (Fig. 7),

Resultant force on plate

$$= CB \text{ bisecting angle } ABD$$

and $\quad CB = 2[(\text{Mass/sec.}) \times \text{Velocity}] \sin 22 \cdot 5°$

$$\therefore \quad \text{Force} = \frac{2 \times 9 \cdot 5 \times 1\,000}{32 \cdot 2} \times 0 \cdot 3827$$

$$= 11 \cdot 89 \text{ lb.}$$

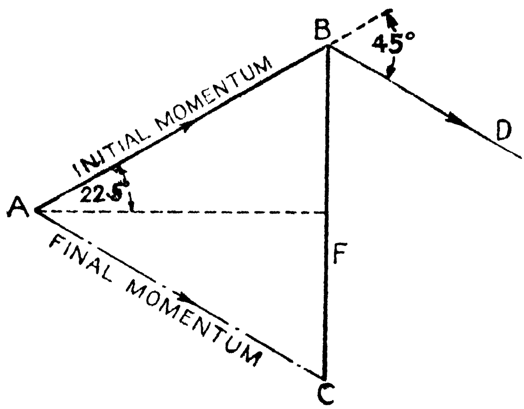

FIG. 7

32. A jet of water, 3 in. diameter, moving at 35 ft./sec., impinges in succession on a series of vanes moving at 25 ft./sec. in the direction of the jet. Each vane is curved through an angle of 130° and the jet strikes each vane tangentially. Find the force exerted by the jet on the vanes. (*W.S.*)

Ans. 64·75 lb.

33. A block of wood of weight 1 000 lb. is mounted on frictionless wheels and is moving at 1 ft./sec. along a horizontal plane. It is overtaken and struck by a bullet weighing 0·1 lb. moving in the same direction. The bullet remains in the block which moves on with a velocity of 1·2 ft./sec. Find the speed of the bullet on striking.

SOLUTION. The momentum lost by bullet = Momentum gained by bullet and block.

Initial momentum of block

$$= (1\,000/g) \times 1 = 1\,000/g \text{ lb. ft. sec. units}$$

Initial momentum of bullet

$$= 0{\cdot}1v/g = v/10g \text{ lb. ft. sec. units}$$

Final momentum of bullet and block

$$= \frac{1\,000{\cdot}1 \times 1{\cdot}2}{g} = \frac{1\,200{\cdot}12}{g} \text{ lb. ft. sec. units}$$

Hence, $1\,200{\cdot}12/g = 1\,000/g + v/10g$

$(1\,200{\cdot}12 - 1\,000) = v/10$

$\therefore \quad v = 10 \times 200{\cdot}12 = 2\,001{\cdot}2$ ft./sec.

34. If in the previous problem the bullet strikes the block at an angle making 45° with the direction of motion of the block and the block is constrained to move only in the original direction, find its velocity after impact and the lateral force which it sustains.

(*Hint:* Vectorial change of momentum.)

Ans. 1·141 ft./sec.
4·39 lb.

35. A motor car has a wheel base of 8 ft. The height of the centre of gravity above the road is 30 in., and the distribution of load when at rest is, front axle 8 cwt., rear axle 12 cwt. If the coefficient of friction between the wheels and the road is 0·35, find the maximum acceleration of the car, (*a*) if the drive is on the front axle, (*b*) if the drive is on the back axle. (*L.U.A.*)

SOLUTION. Taking moments about A (Fig. 8) when at rest we get

$$\bar{x} = (8 \times 12)/20 = 4{\cdot}8 \text{ ft.}$$

(*a*) Drive on front wheels. The maximum force which can be exerted is μR_1, and when accelerating this tends to decrease the pressure on the front axle. Taking moments about B we get,

$$\mu R_1 \times 30/12 + R_1 \times 8 = W(8 - 4{\cdot}8)$$

$$\therefore \quad R_1 = \frac{2\,240 \times 3{\cdot}2}{8{\cdot}875} \text{ and } \mu R_1 = \frac{2\,240 \times 3{\cdot}2 \times 0{\cdot}35}{8{\cdot}875}$$

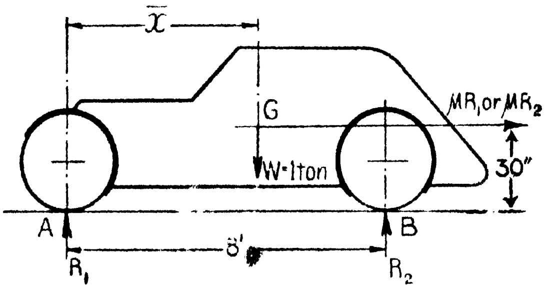

FIG. 8

This latter is the maximum accelerating force and since

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

we have

$$\text{Maximum Acceleration} = \frac{\text{Force}}{\text{Mass}}$$

$$= \frac{2\,240 \times 3{\cdot}2 \times 0{\cdot}35 \times 32{\cdot}2}{8{\cdot}875 \times 2\,240}$$

$$= 4{\cdot}064 \text{ ft./sec.}^2$$

The remainder of the problem is left to the student.

2. CENTRE OF GRAVITY—MOMENT OF INERTIA

Definition. If the mass of every particle of a body be multiplied by its distance from a straight line or a plane, the sum of the products so formed is called the *first moment* of the body about that line or plane.

The *centre of gravity*, or *centre of mass*, of a body is determined by the following relations.

(*a*) *Two-dimensional body* lying in plane XY. The co-ordinates of the centre of gravity are,

$$\bar{x} = \frac{\text{First Moment about } OY}{\text{Mass}} = \frac{\Sigma mx}{\Sigma m}$$

$$\bar{y} = \frac{\text{First Moment about } OX}{\text{Mass}} = \frac{\Sigma my}{\Sigma m}$$

(*b*) *Three-dimensional body.* The co-ordinates of the centre of gravity are,

$$\bar{x} = \frac{\text{First Moment about plane } ZOY}{\text{Mass}} = \frac{\Sigma mx}{\Sigma m}.$$

$$\bar{y} = \frac{\text{First Moment about plane } ZOX}{\text{Mass}} = \frac{\Sigma my}{\Sigma m}.$$

$$\bar{z} = \frac{\text{First Moment about plane } XOY}{\text{Mass}} = \frac{\Sigma mz}{\Sigma m}.$$

In many cases we deal only with two-dimensional bodies or symmetrical bodies which may be treated in the same manner as two-dimensional bodies, by simple artifices. Examples of this will be given. In the cases of bodies with an axis or plane of symmetry it is obvious that this axis or plane must contain the centre of gravity of the body.

1. Find the centre of gravity of the plane lamina shown in Fig. 9.

SOLUTION. Note that for a plane lamina the mass of any area is proportional to that area. Taking the axes OX and OY as shown and dividing the lamina into figures, the positions of the centres of gravity of which are known —in this case rectangles—we have,

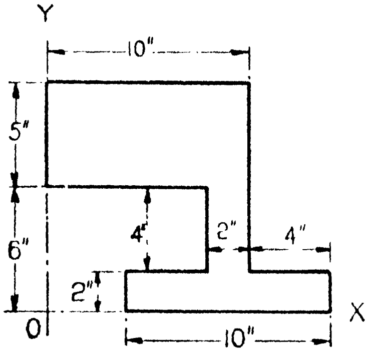

FIG. 9

$$\bar{x} = \frac{\Sigma mx}{\Sigma m} = \frac{\rho[(10 \times 5 \times 5) + (2 \times 4 \times 9) + (10 \times 2 \times 9)]}{\rho[(10 \times 5) + (2 \times 4) + (10 \times 2)]}$$

$$= \frac{502}{78} = 6{\cdot}43 \text{ in.}$$

$$\bar{y} = \frac{\Sigma my}{\Sigma m} = \frac{\rho[(10 \times 5 \times 8{\cdot}5) + (2 \times 4 \times 4) + (10 \times 2 \times 1)]}{\rho[(10 \times 5) + (2 \times 4) + (10 \times 2)]}$$

$$= \frac{477}{78} = 6{\cdot}11 \text{ in.}$$

where ρ = mass of unit area of the material.

2. Find the centre of gravity of a plane semi-circular lamina of radius r (Fig. 10).

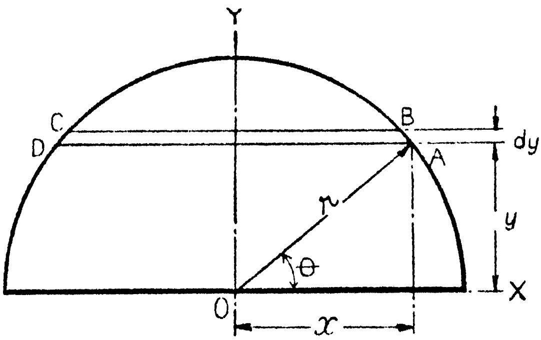

FIG. 10

SOLUTION. We note that in this case there is an axis of symmetry which has been taken as OY. The c.g. lies on

this line. Taking the base as the axis of X and considering an element $ABCD$ of the area,

$$\bar{y} = \frac{\Sigma my}{\Sigma m} = \frac{\Sigma \rho(2xdy)y}{\Sigma \rho(2xdy)} = \frac{2\rho \int xydy}{2\rho \int xdy}.$$

Now $x = r \cos \theta$, $y = r \sin \theta$, $\therefore\ dy = r \cos \theta \,.\, d\theta$.

$$\therefore \qquad \bar{y} = \frac{2\rho \int_0^{\pi/2} r^3 \,.\, \cos^2 \theta \,.\, \sin \theta \,.\, d\theta}{2\rho \int_0^{\pi/2} r^2 \,.\, \cos \theta \,.\, \cos \theta \,.\, d\theta}$$

$$= \frac{2\rho\, r^3 \left[\dfrac{\cos^3 \theta}{3} \right]_0^{\pi/2}}{2\rho\, r^2 \left[\dfrac{\theta}{2} + \dfrac{1}{4} \sin 2\theta \right]_0^{\pi/2}}$$

$$\therefore \qquad \bar{y} = \frac{r^3/3}{\pi r^2/4} = \frac{4r}{3\pi}.$$

3. Show that the c.g. of a right circular solid cone of height h, lies on its axis at a point $h/4$ above the base.

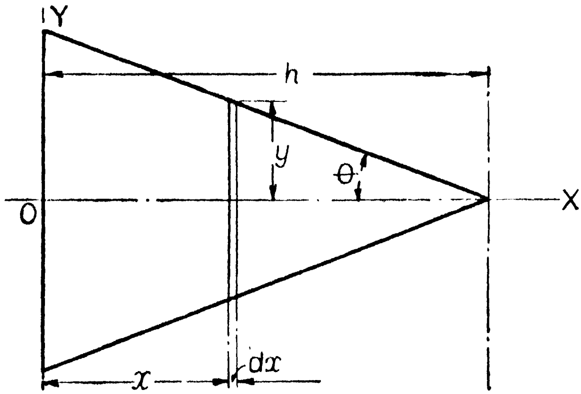

Fig. 11

Solution. Since the cone is symmetrical about its axis the c.g. must lie on this axis. Considering (Fig. 11) an elementary disc distance x from the base and taking the

semi-vertical angle of the cone $= \theta$; let $\rho =$ mass of unit volume of the material of the cone.

Then

Mass of elementary disc

$$= \rho\pi y^2 \,.\, dx.$$

Moment of this mass about OY

$$= \rho\pi y^2 \,.\, x \,.\, dx.$$

$\therefore$ Total moment of cone about OY

$$= \int_{x=0}^{x=h} \rho\pi y^2 x \,.\, dx.$$

Total mass of cone

$$= \int_{x=0}^{x=h} \rho\pi y^2 \,.\, dx.$$

$$\therefore \qquad \bar{x} = \frac{\text{Total moment of cone}}{\text{Total mass of cone}}$$

$$= \frac{\int_{x=0}^{x=h} \rho\pi y^2 \,.\, x \,.\, dx}{\int_{x=0}^{x=h} \rho\pi y^2 \,.\, dx}$$

$$= \frac{\int_{x=0}^{x=h} y^2 \,.\, x \,.\, dx}{\int_{x=0}^{x=h} y^2 \,.\, dx.}.$$

Now $y = (h - x) \,.\, \tan\theta$, and substituting we have,

$$\bar{x} = \frac{\int_0^h (h-x)^2 \,.\, x \,.\, dx}{\int_0^h (h-x)^2 \,.\, dx} = \frac{\left[\frac{h^2x^2}{2} - \frac{2hx^3}{3} + \frac{x^4}{4}\right]_0^h}{\left[h^2x - \frac{2hx^2}{2} + \frac{x^3}{3}\right]_0^h}$$

$$\therefore \quad \bar{x} = \frac{[h^4/2 - \frac{2}{3}h^4 + h^4/4]}{[h^3 - h^3 + h^3/3]} = \frac{h^4/12}{h^3/3} = \frac{h}{4}.$$

4. Find the position of the c.g. of a solid hemisphere of radius r.

Ans. $\frac{3}{8}r$ from base.

5. A right circular cone is made of thin sheet metal, and is bottomless. Find the position of its c.g.

Ans. $\frac{1}{3}$ height from base.

6. A piece of wire is bent into the form of a semicircular arc of radius a (Fig. 12). Show that its centre of gravity lies

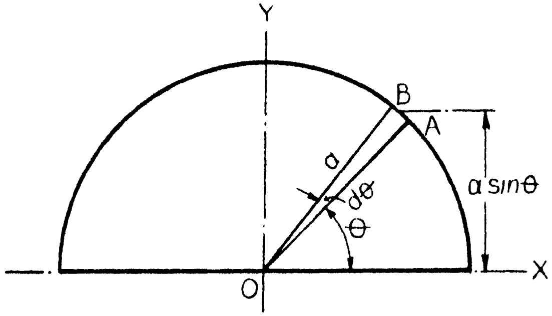

FIG. 12

on the axis of symmetry at a distance from the centre of the circle, given by $h = 2a/\pi$.

SOLUTION. Considering an element AB of the wire and taking ρ = mass of unit length of wire, we have,

Mass of element AB = length $\times\ \rho = \rho a\ .\ d\theta$.

Moment of element about OX

$$= \rho\ .\ a\ .\ d\theta \times a \sin\theta$$
$$= \rho\, a^2\ .\ \sin\theta\ .\ d\theta.$$

Total moment of wire about OX

$$= 2\rho \int_0^{\pi/2} a^2\ .\ \sin\theta\ .\ d\theta.$$

Total mass of wire $= \pi a\ .\ \rho$

$$\text{Hence,}\quad h = \frac{2\rho \int_0^{\pi/2} a^2\ .\ \sin\theta\ .\ d\theta}{\rho\ .\ \pi a} = \frac{2\rho\ a^2 \Big[-\cos\theta\Big]_0^{\pi/2}}{\rho\ .\ \pi a}$$

$$\therefore \qquad h = 2\rho a^2/\rho\pi a = 2a/\pi.$$

7. Find the position of the c.g. of a wire bent into the form of a circular arc of radius a which subtends an angle 2θ at the centre.

Ans. $a \sin \theta/\theta$ from centre on axis of symmetry.

8. A solid right circular cylinder of radius a and length l has a hemispherical depression of the same radius in one end. Find the position of the centre of gravity.

Ans. $\frac{1}{4} . (6l^2 - 8\,al + 3a^2)/(3l-2a)$ from the flat end.

9. A portion of axial length $a/2$ is removed from a solid sphere of radius a. Find the c.g. of the remainder and of the piece removed.

Ans. (i) $\frac{1}{6}a$ from centre.
(ii) $\frac{7}{40}a$ from base.

10. The wheel base of a car (Fig. 13) is b, the radius of each wheel is r, and the total weight of the car is W. When the back wheels rest on a weighing machine table of height h, the machine registers W_1. When the front wheels rest similarly on the machine the latter registers W_2. Determine the position of the centre of gravity of the car. (*W.S.*)

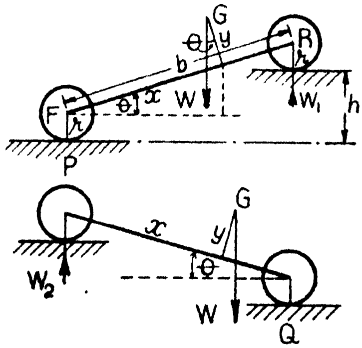

FIG. 13

SOLUTION. Let the c.g. be a distance x from the front axle and height y above the line joining the front and rear axles.

Position (i), taking moments about P,

$$W(x \cos \theta - y \sin \theta) = W_1 b \,.\, \cos \theta \qquad \text{(i)}$$

Position (ii), taking moments about Q,

$$W[(b - x) \cos \theta - y \sin \theta] = W_2 b \,.\, \cos \theta \qquad \text{(ii)}$$

Eliminating y by subtracting (ii) from (i) we get

$$W(2x - b) \cos \theta = b(W_1 - W_2) \cos \theta$$

$$(2x - b) = [(W_1 - W_2)/W] \,.\, b$$

$$\therefore \qquad x = (b/2)[1 + (W_1 - W_2)/W]$$

$$= \tfrac{1}{2} \,.\, b[(W + W_1 - W_2)/W].$$

Substituting this value of x in (i)

$$Wy \sin\theta = \frac{Wb}{2}\left(\frac{W + W_1 - W_2}{W}\right)\cos\theta - W_1 b \cos\theta$$

$$y = (1/W)\,[(b/2)(W + W_1 - W_2) - W_1 b]\,.\cot\theta.$$

Now from the figure

$$\cot\theta = [\sqrt{(b_2 - h^2)}]/h$$

and hence $$y = \tfrac{1}{2}[b\sqrt{(b^2 - h^2)}]/h\,.\,(W - W_1 - W_2)/W.$$

11. A ship's boat, 30 ft. long between hooks A and B at the bow and stern, is supported by ropes of equal length, 6 ft., from davits C and D, 36 ft. apart at the same level. If the hook A lies 1 ft. away from the vertical line passing through C, find the centre of gravity of the boat. (*I.C.E.*)

Ans. On vertical through a point 6 ft. from A on AB.

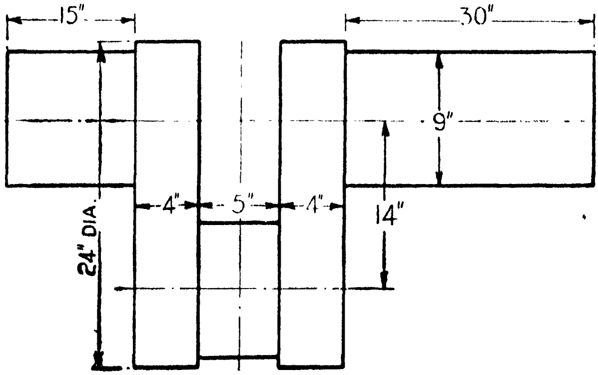

FIG. 14

12. Fig. 14 gives the dimensions of a crankshaft. Find the position of its c.g. with respect to the centre of the crank-pin.

Ans. 4·05 in. to right of pin.
9·62 in. above pin.

13. To find the c.g. of a locomotive one rail is raised a height of 7 in. above the other and it is found that the loads on the upper and lower rails are 25 and 46 tons respectively. If the distance between rails is 5 ft., find the height of the c.g. above rail level.

Ans. 75·5 in.

Definition. If the mass of every particle of a body be multiplied by the square of its distance from a straight line, the sum of these products is called the *second moment* or *moment of inertia* of the body about that line.

Thus if I denotes the moment of inertia of a body about a line,

$$I = \Sigma mr^2$$

where the summation extends over the whole body.

It is often found convenient to express I in the form

$$I = (W/g) \cdot k^2 = \text{Mass} \times k^2$$

The constant k is called the *radius of gyration* of the body about the line considered. Hence we have,

$$(W/g) . k^2 = \Sigma mr^2$$

$$\therefore \quad k^2 = \frac{\Sigma mr^2}{W/g} = \frac{I}{\text{Mass}}.$$

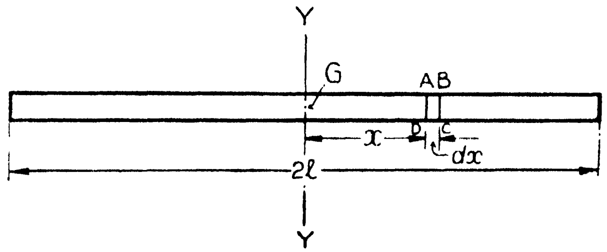

FIG. 15

14. Find the moment of inertia and hence the radius of gyration of a uniform rod of length $2l$ about an axis perpendicular to the rod and through its c.g. (Fig. 15).

SOLUTION. Let w = weight of unit length of the rod ; then considering a small element $ABCD$ of length δx and distance x from the c.g. which is of course at the mid-point of the rod—

Moment of inertia of element about YY

$$= [(w/g)\delta x]x^2$$

$\therefore$ Total moment of inertia of rod about YY

$$= \int_{-l}^{+l} \frac{w}{g} x^2 \, . \, dx$$

$$= (w/g) \left[x^3/3 \right]_{-l}^{+l} = (w/g) \, . \, (2l^3/3).$$

Now the weight of the rod is obviously $2wl$.

Hence, $$k^2 = \frac{I}{W/g} = \frac{\frac{2}{3}(w/g)l^3}{2wl} = \frac{l^2}{3}$$

$\therefore$ $$k = l/\sqrt{3}.$$

15. Find the radius of gyration of a rectangular lamina of sides $2a$ and $2b$, about an axis through its centre lying in its plane perpendicular to the side of length $2a$.

Ans. $k = a/\sqrt{3}$.

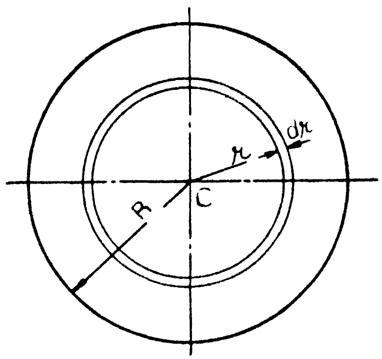

FIG. 16

16. What is the radius of gyration of a circular lamina of radius R about an axis through its centre and perpendicular to its plane? (Fig. 16).

SOLUTION. Let w = weight of unit area and consider an elementary ring of radius r and width δr. Now if δr is very small the moment of inertia of ring about C = $(w/g)(2\pi r \,.\, \delta r)r^2$.

$\therefore$ Total moment of inertia of circle about C

$$= \Sigma\, (w/g)(2\pi r \,.\, \delta r)r^2$$

where the summation is taken over the whole circle. When δr is decreased indefinitely,

$$I_c = \frac{2\pi w}{g}\int_0^R r^3 .\, dr = \frac{2\pi w}{g}\left[\frac{r^4}{4}\right]_0^R$$

$$= (w/g)(\pi R^4/2).$$

The weight of the disc is $W = w \,.\, \pi R^2$.

$\therefore$ $$k^2{}_c = \frac{I_c}{W/g} = \frac{(w/g)(\pi R^4/2)}{(w/g)\pi R^2} = \frac{R^2}{2}$$

$$k_c = R/\sqrt{2}.$$

17. Find the radius of gyration of the disc in Problem 16 about a diameter.

Ans. $k = R/2$.

18. A right-angled uniform triangular plate (Fig. 17) has a base of length a and a height b. Find its radius of gyration about the base.

SOLUTION. Considering the elementary area shown in the diagram and letting w = weight of unit area—

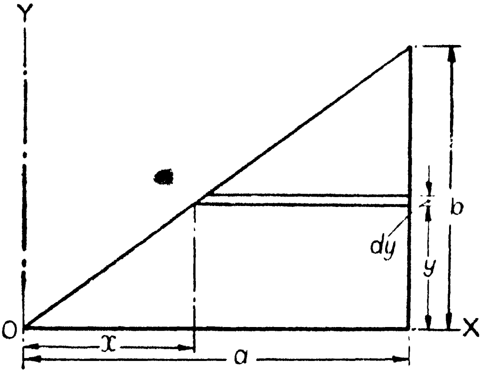

FIG. 17

Moment of inertia of elementary area about the base OX is,

$$(w/g)(a-x)dy \, . \, y^2$$

$$\therefore \qquad I_{OX} = \int_{y=0}^{y=b} (w/g)(a-x)y^2 . \, dy$$

Now from the figure, $x/y = a/b$. $\therefore$ $x = (a/b)y$ and substituting this value of x in the expression for I_{OX} we have,

$$I_{OX} = \frac{w}{g}\int_0^b \left(a - \frac{a}{b}y\right)y^2 . \, dy \; . = \frac{w}{g}\int_0^b a\left(y^2 - \frac{y^3}{b}\right) . \, dy$$

$$= \frac{wa}{g}\,{}_0^b\!\left[\frac{y^3}{3} - \frac{y^4}{4b}\right] = \frac{wa}{g}\left(\frac{b^3}{3} - \frac{b^3}{4}\right)$$

$$= (w/g)(ab^3/12).$$

The weight of the plate is $W = w \, . \, (ab/2)$

$$\therefore \qquad k^2_{OX} = \frac{I_{OX}}{W/g} = \frac{(w/g)(ab^3/12)}{(w/g)(ab/2)} = \frac{b^2}{6}$$

$$k_{OX} = b/\sqrt{6}.$$

19. Find the radius of gyration of a solid sphere of radius R, about a diameter (Fig. 18).

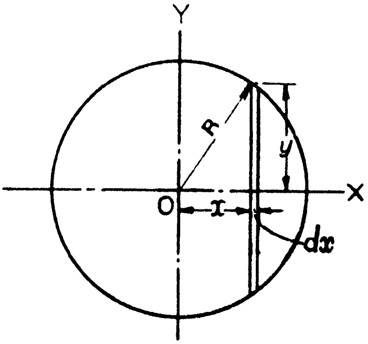

FIG. 18

SOLUTION. Taking w as the weight of unit volume of the material of the sphere and considering an elementary disc of thickness δx and radius $y = \sqrt{(R^2 - x^2)}$—

Moment of inertia of disc about $OX = (w/g)(\pi y^4/2) \cdot \delta x$ by Problem 16. Hence when δx is indefinitely decreased we have,

Total moment of inertia of sphere about OX

$$I_{ox} = \int_{x=-R}^{x=+R} \frac{w}{g} \cdot \frac{\pi y^4}{2} \cdot dx = \frac{w}{g} \cdot \frac{\pi}{2} \int_{-R}^{+R} (R^2 - x^2)^2 . dx$$

$$= \frac{w}{g} \cdot \frac{\pi}{2} \int_{-R}^{+R} (R^4 - 2R^2x^2 + x^4) dx$$

$$= \frac{w}{g} \cdot \frac{\pi}{2} \Big[R^4 x - \frac{2R^2x^3}{3} + \frac{x^5}{5} \Big]_{-R}^{+R}$$

$$= (8/15)(w/g)\pi R^5.$$

Now the total weight of the sphere is

$$w \times \text{volume} = w \cdot 4/3 \cdot \pi R^3 = W$$

$$\therefore \qquad k^2_{ox} = \frac{I_{ox}}{W/g} = \frac{(8/15)(w/g)\pi R^5}{(w/g)(4/3)\pi R^3} = \frac{2}{5} \cdot R^2$$

$$k_{ox} = \sqrt{(2/5)} \cdot R.$$

20. Find the radius of gyration of a homogeneous right circular cone about its central axis, if the radius of its base is R and its height h.

(*Hint:* Divide the cone into elementary discs.)

Ans. $k = (\sqrt{\tfrac{3}{10}})R.$

21. Find the moment of inertia of a uniform rectangular plate of sides $2a$ and $2b$, about one of the sides of length $2b$. Hence, show that if this moment of inertia is denoted by I_{BB}, and W is the weight of the whole plate,

$$I_{BB} = I_{XX} + (W/g) \cdot a^2$$

where I_{XX} = moment of inertia about an axis parallel to the side $2b$ and through the centre of gravity.

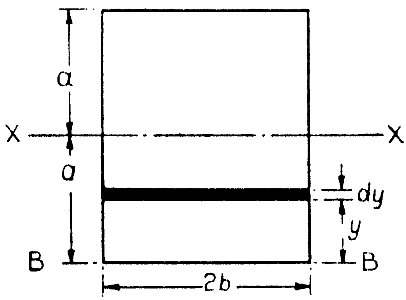

FIG. 19

SOLUTION. Considering the elementary shaded area in Fig. 19—

I_{BB} of element $(w/g) \cdot (2b \cdot \delta y)y^2$ where w = weight of unit area.

Summing up for the whole plate,

$$I_{BB} = \int_0^{2a} \frac{w}{g} \cdot 2b \cdot y^2 \cdot dy \cdot = \frac{w}{g} \cdot 2b \left[\frac{y^3}{3}\right]_0^{2a} = \frac{16}{3} \cdot \frac{w}{g} \cdot a^3 b.$$

Since the weight of the plate is $W = 4w \cdot ab$,

$$I_{BB} = (4/3)(W/g)a^2.$$

The moment of inertia about XX may be found in a similar manner or from Problem 15,

$$I_{XX} = (W/g)(a^2/3)$$

$$\therefore \quad I_{XX} + (W/g)a^2 = (W/g)(a^2/3) + (W/g)a^2 = (W/g)(4/3)a^2$$
$$= I_{BB}.$$

Rule. The result of Problem 21 may be summed up in a general rule which will be found exceedingly useful in calculating moments of inertia, viz.

The moment of inertia of a body about any axis is equal to the moment of inertia about a parallel axis through the centre of gravity together with the product of the mass of the body and the square of the distance from the given axis.

For bodies which have axes of symmetry there exists a simple rule for finding the moment of inertia about any one of these axes. It is known as *Routh's Rule* after its originator.

Moment of inertia about an axis of symmetry

$$= \text{Mass}\left\{\frac{\text{Sum of squares of perpendicular semi-axes}}{3,\ 4\ \text{or}\ 5}\right\}$$

The denominator is to be 3, 4 or 5 according as the body is rectangular, elliptical, or ellipsoidal.

To illustrate the use of this rule take a number of examples, thus—

(i) A uniform rectangular plate of sides $2a$ and $2b$:

I about axis through c.g. parallel to $2a$

$$= (W/g)(b^2/3)$$

I about axis through c.g. perpendicular to plane

$$= (W/g) \,.\, (a^2 + b^2)/3.$$

(ii) A uniform elliptical plate of semi-axes a and b:

I about major axis $2a = (W/g)(b^2/4)$

I about minor axis $2b = (W/g)(a^2/4)$

I about axis through c.g. normal to plane

$$= (W/g) \,.\, (a^2 + b^2)/4.$$

(iii) A homogeneous ellipsoid of semi-axes a, b and c:

I about axis $2a = (W/g) \,.\, (b^2 + c^2)/5$

I about axis $2b = (W/g) \,.\, (c^2 + a^2)/5$

I about axis $2c = (W/g) \,.\, (a^2 + b^2)/5.$

Since the sphere is a particular case of the ellipsoid in which $a = b = c$, we have for a sphere,

I about a diameter $= \frac{2}{5}(W/g)a^2$ where a is the radius.

22. Find the radii of gyration of a hollow sphere of external radius R and internal radius r about,

(i) a diameter,

(ii) a tangent to the outer surface.

Ans. (i) $k^2 = \frac{2}{5}[(R^5 - r^5)/(R^3 - r^3)]$

(ii) $k^2 = \frac{2}{5}[(R^5 - r^5)/(R^3 - r^3)] + R^2$

23. Find the radius of gyration of a homogeneous cone of base radius R and height h about a diameter of the base.

$$Ans.\ k = \sqrt{\left[\frac{3R^2 + 2h^2}{20}\right]}.$$

24. Show that the moment of inertia of the uniform triangular plate in Fig. 20, about the line AY, is given by,

$$I_{AY} = \tfrac{1}{6}(W/g)(m^2 + mn + n^2)$$

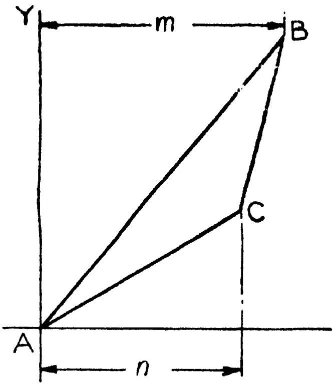

FIG. 20

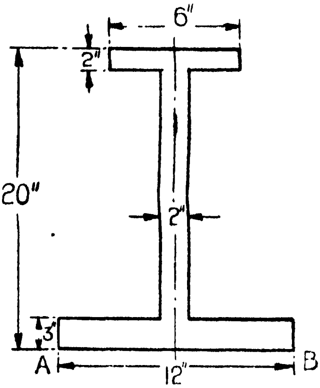

FIG. 21

25. Find the radius of gyration of a uniform semicircular plate about its base given that the radius is R.

Ans. $k = R/2$.

26. A cast iron frame has the section shown in the Fig. 21. Find: (i) the centre of gravity; (ii) the second moment (moment of inertia) about an axis through the centre of gravity and parallel to AB. (*I.Mech.E.*)

Ans. (i) 7·65 in. above AB.
(ii) 3 744 in.^{4}

27. An eccentric sheave consists of a cylinder 10 in. dia. and 2 in. thick having a hole 3 in. dia. The axis of the hole is 3 in. from the centre of the disc. If the material of the eccentric weighs 0·26 lb./per in.^{3}, find its moment of inertia about the axis of the hole. *Ans.* 873·8 lb. in.^{2}

28. Find the radius of gyration of the crankshaft of Problem 12 about its axis. *Ans.* 8·85 in.

3. COUPLE—ANGULAR MOMENTUM—ENERGY—DYNAMICS OF SIMPLE MECHANISMS

In linear motion we have the relation,

Force = Rate of change of momentum

$= (d/dt)(Mv) =$ Mass $\times$ Acceleration.

Similarly for rotational motion we have,

Couple = Rate of change of angular momentum

$= (d/dt)(I\omega) =$ Moment of inertia $\times$ Angular Acceleration.

1. A uniform bar (Fig. 22) of length 5 ft. and weight 15 lb. is fixed horizontally at one end to a vertical shaft

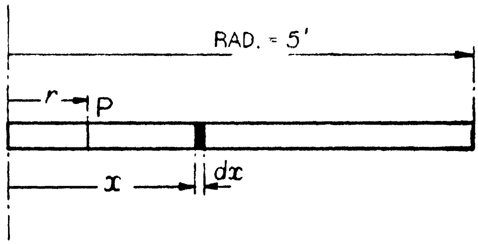

FIG. 22

which rotates at 150 r.p.m. Find the maximum tension in the bar due to rotation. If the bar is then brought to rest in 5 sec. by a constant couple applied to the shaft, find the bending moment at the axis of rotation.

SOLUTION. (i) The tension in the bar when rotating uniformly is due to the centripetal acceleration. Considering an element of the bar of length dx and distance x from the axis of rotation—

Tension at P due to rotation of this element

$$= (15/5)(dx/g)\omega^2 . x$$

where $\omega =$ angular velocity of the bar

$$= (2\pi \times 150)/60 = 5\pi \text{ radn./sec.}$$

Hence total tension at P due to rotation

$$T = \frac{15}{5}(5\pi)^2 \cdot \frac{1}{g}\int_r^5 x\,dx = \frac{75\pi^2}{g}\left[\frac{x^2}{2}\right]_r^5 = \frac{75\pi^2}{2g}(25 - r^2)$$

This is evidently a maximum where $r = 0$, that is on the axis of rotation.

$$\therefore \qquad T_{max} = \frac{75\pi^2 \times 25}{2 \times 32{\cdot}2} = 287{\cdot}2 \text{ lb.}$$

(ii) The bending moment at the axis due to the angular retardation of the bar is simply the couple required to produce the retardation.

Now $\omega_{final} = \omega_{initial} - \dot{\omega}t$, where $\dot{\omega}$ = angular retardation

and since $\qquad t = 5$ sec. we have

$$0 = 5\pi - 5\dot{\omega}$$

$$\therefore \qquad \dot{\omega} = \pi \text{ radn./sec.}^2$$

Hence,

$$\begin{aligned} \text{Couple} &= \text{Moment of inertia} \times \text{Angular retardation} \\ &= \frac{15}{32{\cdot}2} \times \left(\frac{25}{3}\right) \times 3{\cdot}14 \\ &= 12{\cdot}19 \text{ lb. ft.} \\ &= \text{required bending moment.} \end{aligned}$$

2. A cage weighing 1·5 tons is raised by means of a rope coiled round a drum of 5 ft. dia. mounted on a horizontal shaft. The drum and shaft weigh 2 000 lb. and the radius of gyration is 28 in.

A motor supplies a constant torque of 9 000 lb. ft. to the shaft. Assuming no friction and that the rope is tight when the motor begins to revolve, find

(*a*) the acceleration of the cage,
(*b*) the time required to raise it 40 ft. from rest,
(*c*) the tension of the rope.

(*W.S.*)

SOLUTION. (*a*) For the shaft and drum,

Couple = Moment of inertia × Angular acceleration

$$\therefore \qquad \left(9\,000 - \frac{5}{2}\mathrm{T}\right) = \frac{2\,000}{32{\cdot}2}\left(\frac{28}{12}\right)^2 . \dot{\omega}$$

where T is the tension in the rope and $\dot{\omega}$ the angular acceleration.

$$\text{Simplifying,} \qquad 9\,000 - \frac{5}{2} . \mathrm{T} = 388{\cdot}2 . \dot{\omega}. \qquad \text{(i)}$$

If f = linear acceleration of the cage, we have,

$$f = (5/2)\dot{\omega},$$

and since

Force = Mass × Acceleration,

we have considering the cage,

$$\mathrm{T} - (2\,240 \times 1{\cdot}5) = \frac{2\,240 \times 1{\cdot}5}{32{\cdot}2} \times \frac{5}{2}\dot{\omega}$$

or

$$\mathrm{T} - 3\,360 = 260{\cdot}8\dot{\omega}$$

$$\therefore \qquad \mathrm{T} = 260{\cdot}8\dot{\omega} + 3\,360. \qquad \text{(ii)}$$

Substituting this value of T in (i) we get,

$$9\,000 - 652\dot{\omega} - 8\,400 = 388{\cdot}2\dot{\omega}$$

$$\therefore \qquad \dot{\omega} = \frac{600}{1\,040{\cdot}2} \text{ radn./sec.}^2$$

Acceleration of cage $= (5/2)\dot{\omega} = 1{\cdot}442$ ft./sec.^{2}

(*b*) Since this acceleration is constant we have,

$$40 = \tfrac{1}{2} . ft^2 = \tfrac{1}{2} \times 1{\cdot}442 \times t^2$$

$$\therefore \qquad t = \sqrt{\frac{80}{1{\cdot}442}} = 7{\cdot}447 \text{ sec.}$$

(*c*) The tension in the rope from (ii) above, is

$$\mathrm{T} = \frac{260{\cdot}8 \times 600}{1\,040{\cdot}2} + 3\,360 = 3\,510{\cdot}5 \text{ lb.}$$

3. Find the torque which must be applied to the shaft in Problem 2 in order that the cage may descend at a uniform speed of 2 ft./sec. (*W.S.*)

Ans. 7 829 lb. ft.

4. A drum weighing 2 tons and having a radius of gyration of 2 ft. supports a weight of 500 lb. by means of a rope which is wound round the drum on a diameter of 6 ft. If the system is started from rest, find the velocity of the weight and the tension in the rope after the weight has fallen 6 ft.

Ans. 8·806 ft./sec.
399·6 lb.

5. A flywheel weighs 5 tons and has a radius of gyration of 3 ft. When it is spinning at 2 500 r.p.m. the power is removed and it is found to slow down under constant friction, to 2 100 r.p.m. in 4 min. Find the friction couple, and assuming this independent of speed find the time taken to come to rest from the initial speed.

SOLUTION.

Couple = Moment of inertia × Angular acceleration.

If ω_1 = initial angular velocity

and ω_2 = angular velocity at time t, we have

$$\omega_2 = \omega_1 - \dot{\omega}t, \text{ where } \dot{\omega} = \text{angular retardation.}$$

$$\therefore \quad 2\,100 = 2\,500 - \dot{\omega} \times 4$$

$$\dot{\omega} = 400/4 = 100 \text{ rev./min.}^2$$

$$= \left(\frac{2\pi \times 100}{3\,600}\right) \text{radn./sec.}^2$$

Hence

$$\text{Friction couple} = \frac{5 \times 2\,240}{32{\cdot}2} \times (3)^2 \times \frac{2\pi \times 100}{3\,600}$$

$$= 546 \text{ lb. ft.}$$

To find the time to come to rest we have $\omega_2 = 0$ and consequently,

$$0 = 2\,500 - 100t$$

$$\therefore \quad t = 2\,500/100 = 25 \text{ min.}$$

6. The cage of a goods hoist weighs 9 cwt. and carries a maximum load of 15 cwt. It is raised by a rope passing over a 4 ft. dia. drum of weight 800 lb. and radius of gyration 18 in. The other end of the rope is connected to a balance weight, the cage being overbalanced when empty to the extent of 40% of the full load. If the drive, when raising the maximum load is to be capable of a performance equivalent to an acceleration of 4 ft./sec.^{2} at a speed of 10 ft./sec., calculate the drum torque and the power necessary for the masses given. *(L.U.)*

Ans. 2 506·8 lb. ft.
25·72 h.p.

7. A fly-wheel weighing 5 tons and having a radius of gyration of 3 ft. is mounted on a shaft 8 in. dia. If a brake is applied to the shaft so that it is brought to rest from a speed of 100 r.p.m. in 30 sec., what will be the shearing force on the keys securing the wheel to the shaft? *(I.C.E.)*

Ans. 0·73 ton.

8. A wire rope is fixed to and coiled round a drum 8 ft. dia. and the free end supports a load of 3 tons. If the weight of the drum is $1\frac{1}{2}$ tons and its radius of gyration is $3\frac{1}{2}$ ft., find the velocity of the load after it has fallen 100 ft. from rest. The wire rope weighs 5 lb. per ft. length.

Ans. 75 ft./sec.

9. Define angular momentum, and derive an expression for the angular momentum of a fly-wheel of radius of gyration k and weight W tons revolving at N r.p.m.

Show that the rate of change of angular momentum of such a fly-wheel is equal to the applied torque.

A motor gives 10 h.p. at 700 r.p.m. On the shaft is a fly-wheel weighing 1 ton and having a radius of gyration of 1·5 ft.

Assuming that the torque of the motor is constant, find the time in which the motor, starting from rest, will get up a speed of 700 r.p.m. *(I.Mech.E.)*

Ans. 2 min. 33 sec.

10. A body moves on a smooth horizontal table in a circle 4 ft. dia. at a constant speed of 12 ft./sec., and is controlled by a string 2 ft. long pinned to the table at a point *C*. The string is suddenly lengthened to 3 ft., being still pinned at *C*, and the body ultimately describes a circle 6 ft. dia.

Assuming the string to be absolutely unstretchable and unbreakable, describe the intermediate stages of the motion. The weight of the body being 5 lb., show by means of diagrams the horizontal forces acting on, and the velocities of, the body at the several important stages.

Find the sudden change of momentum which occurs when the string tightens, and determine the constant force which, acting for a time equal to that of the free motion of the body, would produce an equal change of momentum. (*W.S.*)

11. A fly-wheel and shaft of weight $\frac{1}{2}$ ton and radius of gyration 3 ft., are rotating in fixed bearings at 300 r.p.m. By means of a clutch on the shaft another similar system, of wheel and shaft weight 1 ton and radius of gyration 4 ft., is suddenly connected to the first shaft. Find the common speed of rotation.

Solution. Since there are no frictional or other losses the total angular momentum of the system must remain constant.

Initial angular momentum $I\omega$

$$= \frac{2\,240 \times (3)^2}{2 \times 32{\cdot}2}\left(\frac{2\pi \times 300}{60}\right)$$

$$= 9\,828 \text{ lb. ft. sec. units.}$$

Final angular momentum $= (I + I_1)\omega_1$

$$= \left[\frac{2\,240}{2} \times (3)^2 + 2\,240 \times (4)^2\right]\frac{1}{32{\cdot}2}\cdot\frac{2\pi N_1}{60}$$

$$= 149{\cdot}3N_1 \text{ lb. ft. sec. units.}$$

Since Initial angular momentum = Final angular momentum,

$$9\,828 = 149{\cdot}3N_1$$

$$\therefore \quad N_1 = 65{\cdot}85 \text{ r.p.m.}$$

12. A motor cycle of total weight 350 lb. is at rest with the clutch withdrawn and the engine running light at 1 000 r.p.m. The total moment of inertia of the rotating parts of the engine is 25 lb. ft.^{2}, and the road wheels have a diameter of 26 in. If low gear is engaged, and is such that the engine makes 12 revolutions for one revolution of the road wheels, and the clutch is suddenly let in, find the initial forward velocity of the motor cycle.

SOLUTION. Angular momentum of rotating parts $= I\omega$.

$$= \frac{25 \times 2\pi \times 1\,000}{32{\cdot}2 \times 60} = 81{\cdot}27 \text{ units.}$$

Let V = initial velocity of cycle in ft./sec., then after the impulse the angular velocity of the engine is

$$\omega_1 = \frac{12}{13}V \times 12 = \frac{144}{13}V$$

and the angular momentum is then

$$I\omega_1 = \frac{25}{32{\cdot}2} \times \frac{144}{13}V = 8{\cdot}6V \text{ units.}$$

Hence,

$$\text{Change of angular momentum} = (81{\cdot}27 - 8{\cdot}6V).$$

This change imparts an impulsive couple at the road wheels, given by

$$\frac{F \times 13}{12 \times 12} = (81{\cdot}27 - 8{\cdot}6V)$$

$$\therefore \qquad F = \frac{144}{13}(81{\cdot}27 - 8{\cdot}6V) \text{ lb.}$$

where F is the impulsive force between road and wheels. Equating this force to the instantaneous change of linear momentum of the cycle, we get,

$$(144/13)\,(81{\cdot}27 - 8{\cdot}6V) = (350/32{\cdot}2)V$$

$$\therefore \qquad V = 4{\cdot}42 \text{ ft./sec.}$$

13. Fig. 23 represents a tilt hammer hinged at *A* and raised 30° ready to strike an object *B*. If the total mass of

the hammer is 10 lb., the distance of its centre of gravity G from A is 2 ft., and its radius of gyration about the axis of the hinge is 2·2 ft., calculate the force of the blow on B which may be assumed to take place in 0·004 sec. Also calculate the reaction at the hinge. (*L.U.A.*)

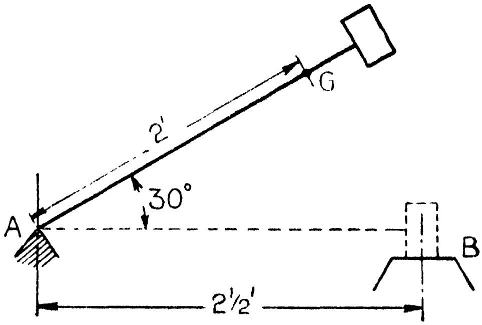

FIG. 23

SOLUTION. We must first find the angular velocity at the instant of striking, by equating the loss of potential energy to the gain of kinetic energy, thus:

$$W \times (AG) \sin 30^\circ = \tfrac{1}{2} I\omega^2$$

$$10 \times 2 \times \tfrac{1}{2} = \tfrac{1}{2} \cdot (10/32{\cdot}2)(2{\cdot}2)^2 \cdot \omega^2$$

$$\therefore \quad \omega = (1/2{\cdot}2)\sqrt{(2 \times 32{\cdot}2)} = 3{\cdot}648 \text{ radn./sec.}$$

Hence the change of angular momentum caused by the blow is

$$I\omega = (10/32{\cdot}2) \times (2{\cdot}2)^2 \times 3{\cdot}648 = 5{\cdot}481 \text{ units.}$$

This change occurs in time 0·004 sec., and is due to a force P at B.

$$\therefore \quad P \times 2{\cdot}5 \times 0{\cdot}004 = 5{\cdot}481$$

$$P = \frac{5{\cdot}481}{2{\cdot}5 \times 0{\cdot}004} = 548{\cdot}1 \text{ lb.}$$

Let R be the impulsive reaction at A. At the instant of impact the point G is moving with a velocity

$$\omega \cdot (AG) = 2 \times 3{\cdot}648 = 7{\cdot}296 \text{ ft./sec.}$$

Momentum of mass supposed concentrated at G,

$$= (10/32{\cdot}2) \times 7{\cdot}296 = 2{\cdot}266 \text{ units.}$$

This is destroyed in 0·004 sec. and hence

$$Pt - R = \text{Change of momentum} = 2{\cdot}266$$

$$\therefore \quad (548{\cdot}1 \times 0{\cdot}004) - R = 2{\cdot}266$$

$$R = -\,0{\cdot}074 \text{ lb.}$$

Since R was assumed in the opposite sense to P, the negative sign means that it is actually acting in the same sense.

14. A wheel rotates freely about its axis, which is horizontal, at a mean speed of 300 r.p.m. The weight of the wheel is 90 lb., its radius of gyration is 6 in. and its mass centre is 0·05 in. from the axis of rotation.

Find the percentage fluctuation of speed from the mean value.

If a brake is applied to the axle which is 3 in. dia., the coefficient of friction being 0·2, find the constant pressure of the brake that will stop the wheel in 12 sec. (*W.S.*)

Ans. 3·3%
73·16 lb.

15. A fly-wheel (Fig. 24), whose axis is vertical, has a moment of inertia A, and the radius of its axle is a. A

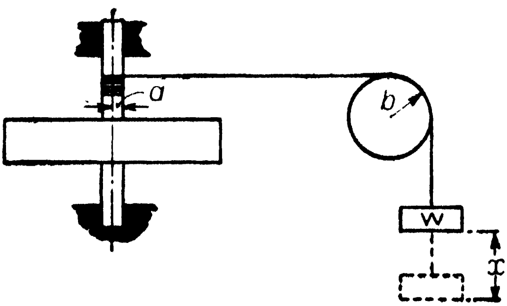

FIG. 24

string is wound round the axle, and passes horizontally to a pulley, the radius of the pulley being b, and its moment of inertia B. At the end of the vertical portion of the string is suspended a weight. If the system starts from rest, find the angular velocities of the fly-wheel and the pulley and the linear velocity of the weight, in terms of the distance

through which the weight has fallen, the friction of the bearings of the fly-wheel and pulley being neglected.

Prove that the ratio of the tension in the horizontal portion of the string to that in the vertical portion is

$$Ab^2/(Ab^2 + Ba^2) \qquad (W.S.S.)$$

SOLUTION. Let ω = angular velocity of fly-wheel. Then ωa = linear velocity of weight W, and $\omega a/b$ = angular velocity of pulley. In this example we will make use of the principle of conservation of energy, in the form,

Total energy of system = constant

or

Loss of potential energy of weight

= Gain of kinetic energy of system

The k.e. of a rotating body is $\frac{1}{2}I\omega^2$. The k.e. of a body moving with linear velocity v is $\frac{1}{2}Mv^2$. Hence we have, if x is the distance moved by the weight W,

$$Wx = \tfrac{1}{2}A\omega^2 + \tfrac{1}{2}B(\omega a/b)^2 + \tfrac{1}{2}(W/g)(\omega a)^2$$

$$Wx = \tfrac{1}{2}\omega^2[A + (a^2/b^2)B + (W/g)a^2]$$

$$\therefore \qquad \omega = \sqrt{\frac{2Wx}{[A + (a^2/b^2)B + (W/g)a^2)]}}$$

= Angular velocity of fly-wheel.

and

$$\omega\frac{a}{b} = \frac{a}{b}\sqrt{\frac{2Wx}{[A + (a^2/b^2)B + (W/g)a^2]}}$$

= Angular velocity of pulley

and

$$\omega a = a\sqrt{\frac{2Wx}{[A + (a^2/b^2)B + (W/g)a^2]}}$$

= Linear velocity of weight.

If $\qquad \dot{\omega}$ = angular acceleration of the fly-wheel

Then $\qquad \dot{\omega}a$ = linear acceleration of the weight

and $\qquad \dot{\omega}a/b$ = angular acceleration of the pulley.

If T_H and T_V are the horizontal and vertical tensions respectively, we have

(i) $T_H \times a = A\dot{\omega} \quad \therefore T_H = (A/a)\dot{\omega}$.

(ii) $(T_V - T_H)b = B\dot{\omega} \,.\, (a/b) \quad \therefore (T_V - T_H) = B(a/b^2).\dot{\omega}$.

By division,

$$(T_V - T_H)/T_H = (B/A)(a^2/b^2)$$

$$T_V/T_H - 1 = (B/A)(a^2/b^2)$$

$$T_V/T_H = 1 + (B/A)(a^2/b^2) = (Ab^2 + Ba^2)/Ab^2$$

$$\therefore \quad T_H/T_V = Ab^2/(Ab^2 + Ba^2).$$

16. A motor car is geared so that for 1 revolution of the engine the car travels 1·2 ft. along the road. The mass of the car has a certain fly-wheel effect in steadying the running of the engine, and if the engine is to be run separately for test purposes a fly-wheel should be added to give the same steadying effect as the car. Calculate the moment of inertia of a fly-wheel to replace the car if the latter weighs 1 200 lb. State the weight of the wheel if it may be treated as a ring concentrated at a radius of 6 in. (*W.S.*)

Solution. Let the angular velocity of the engine be ω radn./sec.

Then Linear speed of car $= (\omega/2\pi) \times 1{\cdot}2$ ft./sec.

At this speed the kinetic energy of the car is

$$\text{K.e.} = \frac{1}{2} \times \frac{1\,200}{32{\cdot}2}\left(\frac{1{\cdot}2\omega}{2\pi}\right)^2$$

Let I be the moment of inertia of the necessary fly-wheel; then the k.e. of this fly-wheel must be the same as that of the car,

$$\text{K.e.} = \tfrac{1}{2} \,.\, I\omega^2.$$

And so,

$$\frac{1}{2} \,.\, I\omega^2 = \frac{1}{2} \cdot \frac{1\,200}{32{\cdot}2} \cdot \left(\frac{1{\cdot}2}{2\pi}\right)^2 \cdot \omega^2$$

$$I = \frac{1\,200}{32{\cdot}2} \cdot \left(\frac{1{\cdot}2}{2\pi}\right)^2$$

$$\therefore \quad I = 1{\cdot}36 \text{ lb. ft.}^2 \text{ units.}$$

Now $(W/g)k^2 = I = 1{\cdot}36$, and $k = \frac{1}{2}$. ft.

$$\therefore \quad \frac{W}{32{\cdot}2} \cdot \frac{1}{4} = 1{\cdot}36$$

$$\therefore \quad W = 175{\cdot}2 \text{ lb.}$$

17. An engine runs at 180 r.p.m. against a constant resisting torque of 1 270 lb. ft. The fly-wheel weighs $\frac{1}{2}$ ton and its radius of gyration is 1·11 ft. For a certain angular interval of 62° the crank effort has a mean value of 1 440 lb. ft. and reaches a maximum of 1 630 lb. ft. Find

(*a*) The effective horse-power of the engine and the kinetic energy of the fly-wheel at the speed of 180 r.p.m.

(*b*) The percentage increase of speed during the interval under consideration.

(*c*) The maximum angular acceleration which occurs during this interval. (*W.S.*)

Ans. (*a*) 43·5 and 7 607 units.
(*b*) 1·17%
(*c*) 8·4 radn./sec.^{2}

18. The reciprocating parts of a horizontal engine weigh 500 lb. The engine has a stroke of 2 ft., and the connecting rod is "long." At $\frac{1}{4}$ stroke there is an effective total pressure on the piston of 30 000 lb. What power is being delivered to the piston at this instant if the engine runs at 120 r.p.m.?

How much of this power is being transmitted to the crank-pin? (*W.S.*)

(*Hint:* Since the connecting rod is "long," the piston moves with simple harmonic motion.)

Ans. 593 h.p.; 0·96 of total.

19. A railway truck has four wheels each weighing 500 lb. and having a radius of gyration of 18 in. and outer diameter of 3 ft. 6 in. The additional weight of the truck and its load is $2\frac{1}{2}$ tons.

If the truck starts from rest down an incline of 1 in 100, how long will it take to cover 150 yd., on the assumption that wind resistance and friction are equivalent to a constant force of 50 lb., and that the wheels do not slip?

SOLUTION. The moment of inertia of each wheel is

$$\frac{500}{32\cdot2}\cdot\left(\frac{3}{2}\right)^2 = 34\cdot93 \text{ lb. ft.}^2 \text{ units.}$$

Considering the energy of the system we have

(Loss of potential energy) − (work done against resistance)
= (Gain of kinetic energy)

In moving 150 yd.,

$$\text{Loss of p.e.} = \frac{450}{100} \times 7\,600 = 34\,200 \text{ ft. lb.}$$

$$\text{Work done against resistance} = 50 \times 450$$
$$= 22\,500 \text{ ft. lb.}$$

If V = velocity of truck in ft./sec. after 150 yd., then $4V/7$ = angular velocity of wheels in radn./sec. after 150 yd. Hence total k.e. of truck and wheels is,

$$\tfrac{1}{2}(4 \times 34\cdot93)(4V/7)^2 + \tfrac{1}{2}(7\,600/32\cdot2)\,.\,V^2 = 140\cdot81V^2.$$

Substituting in the energy equation,

$$34\,200 - 22\,500 = 140\cdot81V^2$$

$$\therefore \quad V = 9\cdot116 \text{ ft./sec.}$$

Since this speed is attained from rest under the action of a constant force we have,

$$V^2 = 2fs$$

$$(9\cdot116)^2 = 2 \times 450 \times f$$

$$\therefore \quad f = 0\cdot0923 \text{ ft./sec.}^2$$

And hence the time required is given by,

$$V = ft.$$

$$9\cdot116 = 0\cdot0923t$$

$$\therefore \quad t = 98\cdot72 \text{ sec.}$$

20. A post is being driven into the ground by blows from a weight which, between successive impacts, is raised an approximately constant height and then allowed to fall under gravity. The post is observed to have penetrated 6 in. after the first blow and 11 in. after the first two blows. Assuming that the resistance to penetration at a depth of x in. is $(A + Bx)$ lb., where A and B are numerical constants, determine the total distance penetrated after three blows and after six blows. (*W.S.S.*)

Ans. 15·38 in. and 26·3 in.

21. A constant horse-power, H, is expended in propelling a body of weight w lb. horizontally against a constant resistance R lb.

Show that the time taken to acquire, from rest, one-half of the greatest velocity is

$$17{\cdot}1(\log_e 2 - \tfrac{1}{2})(wH/R^2) \text{ sec.},$$

and that the distance covered is

$$9394\ (\log_e 2 - \tfrac{5}{8})\ (wH^2/R^3) \text{ ft.} \qquad (W.S.S.)$$

22. A twin-cylinder engine is single acting, with its cranks set at right angles, and it runs at 1 500 r.p.m. The torque-crank angle diagram for each cylinder is practically a triangle for the power stroke, with a maximum torque of 120 ft. lb. at 60° after "in" dead centre of the corresponding crank. The torque on the return stroke is negligible. Find (*a*) the horse-power developed; (*b*) the weight of the fly-wheel, concentrated at 8 in. radius, to keep the speed within $\pm$ 3% of the mean; (*c*) the angle turned through by the crank while it is being speeded up. (*L.U.*)

Ans. (*a*) 17·13 h.p.
(*b*) 8·7 lb.
(*c*) $5\pi/6$ radn.

23. A small, single-cylinder, four-stroke cycle, oil engine, of 5 in. stroke length, develops 7 h.p. at 1 000 r.p.m. The excess energy, delivered during the power stroke, is 78% of the energy per cycle. The engine is fitted with a combined fly-wheel and belt pulley of weight 160 lb. and radius of gyration $7\frac{1}{4}$ in. The rotating parts of the engine—part

connecting rod, crank-pin, etc.—are equivalent to 9 lb. concentrated at the crank radius, and are balanced by weights fixed to the crank webs, the centre of gravity and radius of gyration of which are, respectively, 3 in. and $3\frac{3}{4}$ in.

Estimate the range of speed fluctuation of the engine; and state the percentage error that would be incurred in this estimate by considering the fly-wheel only. (*L.U.*)

Ans. 1·187% fluctuation.
2·36% error.

24. The torque exerted on a crankshaft of an engine when corrected for balance is given by the expression

$$10 + 5 \sin 2\theta - 7 \cos 4\theta,$$

in foot-ton units. Assuming that the resistance is constant, find the moment of inertia of the fly-wheel if the speed variation is not to exceed 0·3% above or below the mean speed, which is 120 r.p.m. (*W.S.S.*)

Ans. 118·2 ton ft.^{2}

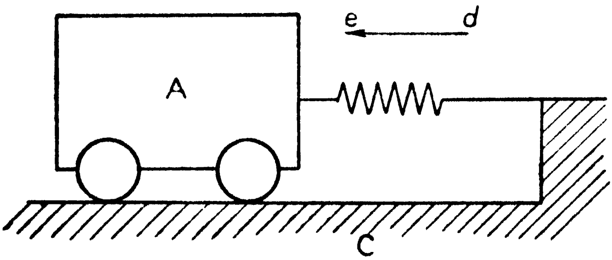

Fig. 25

25. A block of wood *A* (Fig. 25) weighing $1\frac{1}{2}$ kg. is mounted on frictionless wheels, and attached by a light spring balance *B* to one end of a plane *C*. In this position the spring balance is not extended.

A bullet, weighing 1·75 g., is fired from an air-gun into the block in the direction *de*. After firing the block moves along the plane and extends the spring balance until this registers 55 g.; it then moves back towards its first position. In the scale of the balance the 10 g. marks are 12 mm. apart.

Find the velocity of the bullet just before striking the block. (*W.S.*)

SOLUTION. Since there is no change of momentum at impact, we have

$$1\,501{\cdot}75\ V = 1{\cdot}75v$$

$$\therefore \qquad V = (1{\cdot}75/1\,501{\cdot}75)v$$

where V = initial velocity of block and bullet,
v = velocity of bullet.

Initial k.e. of bullet and block

$$= \frac{1}{2} \cdot \frac{1\,501{\cdot}75}{453{\cdot}6 \times 32{\cdot}2} \cdot V^2$$

$$= \frac{1}{2} \cdot \frac{1\,501{\cdot}75}{453{\cdot}6 \times 32{\cdot}2} \left(\frac{1{\cdot}75v}{1\,501{\cdot}75}\right)^2.$$

This must be equated to the work done in stretching the spring.

The spring stretches an amount $\frac{55}{10} \times 12$ mm.

$$= \frac{55 \times 12}{10 \times 2{\cdot}54 \times 10 \times 12}\ \text{ft.}$$

The maximum pull in the spring is 55 g. = 55/453·6 lb.

$\therefore$ Work done in stretching spring

$$= \frac{1}{2} \times \frac{55}{453{\cdot}6} \times \frac{55 \times 12}{100 \times 2{\cdot}54 \times 12}$$

$$= \frac{55 \times 55}{453{\cdot}6 \times 200 \times 2{\cdot}54}\ \text{ft. lb.}$$

Hence we have

$$\frac{1}{2} \times \frac{1\,501{\cdot}75}{453{\cdot}6 \times 32{\cdot}2} \times \left(\frac{1{\cdot}75}{1\,501{\cdot}75}\right)^2 v^2$$

$$= \frac{55 \times 55}{453{\cdot}6 \times 200 \times 2{\cdot}54}$$

$$\therefore \qquad v = 434{\cdot}5\ \text{ft./sec.}$$

26. Two similar loaded trolleys shown in Fig. 26 moving in opposite directions with equal velocity u, collide, impact occurring at the point P. Immediately after impact the

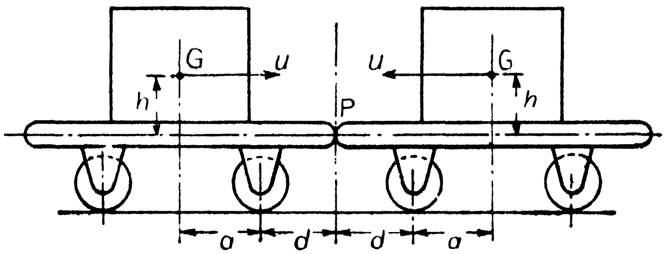

FIG. 26

horizontal velocity of the point of contact is zero, but it has a downward vertical velocity v.

Show that

$$v = dhu/(a^2 + h^2 + k^2)$$

where k is the radius of gyration of each loaded trolley about a horizontal transverse axis through its centre of gravity. The inertia of the wheels may be neglected.

(*Mech. Sc. Tripos Cam.*)

4. SIMPLE HARMONIC MOTION—SMALL OSCILLATIONS

Definition. When a particle moves along a straight line with an acceleration always directed to a fixed point in the line, and proportional to the distance from the particle to the point, then the particle is said to move with simple harmonic motion.

1. A particle describes a circular path of radius r with uniform angular velocity ω. Find the motion of its projection on a diameter.

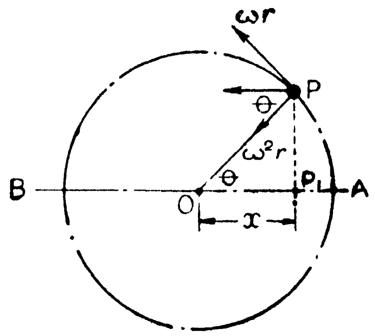

FIG. 27

SOLUTION. Let AB (Fig. 27) be the diameter on which P_1, the projection of P, moves. At time t let θ be the angle which the radius $OP = r$, makes with OA. Let x = displacement of P_1 from O measured along OA.

Then $\qquad x = OP_1 = OP\cos\theta = r\cos\theta$

$\therefore \qquad \cos\theta = x/r.$

The velocity of P_1 is the horizontal component of the velocity of P and is directed along OA. If this velocity is v,

$$v = \omega r\sin\theta = \omega r\sqrt{(1 - x^2/r^2)} = \omega\sqrt{(r^2 - x^2)}$$

The acceleration of P_1 is the horizontal component of the radial acceleration, $\omega^2 r$, of P,

$$f = -\omega^2 r\cos\theta$$
$$= -\omega^2 x \text{ directed towards } O \text{ along } OA.$$

Hence the projection P_1 of P moves along AB with simple harmonic motion.

The fixed point O is called the *centre of oscillation* and the displacement r on each side of O is called the *amplitude of the motion*.

A complete oscillation is from A to B and back to A, and the time, or period, of a complete oscillation is therefore given by

$$t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{x}{\omega^2 x}} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

It follows that any s.h.m. may be treated as the projection on a diameter of uniform motion round a circle whose radius is the amplitude of the s.h.m.

2. A particle starts from rest with s.h.m. at 10 ft. from the centre of its path. If its initial acceleration is 2 ft./sec.^{2} find

(*a*) the periodic time of the motion.

(*b*) the velocity and acceleration 6 ft. from the centre of its path.

SOLUTION. Consider the motion as the projection on a diameter of uniform motion round a circle, as in Problem 1. If the displacement from the centre at any instant be denoted by x,

Acceleration $= \omega^2 . x$

$\therefore$ $2 = \omega^2 \times 10$ initially,

and so $\omega^2 = 2/10$ or $\omega = 1/\sqrt{5}$ radn./sec.

The velocity in any position x, is given by

$$v = \omega\sqrt{(r^2 - x^2)} \text{ where } r \text{ is the amplitude.}$$

Now $v = 0$ in the extreme positions where $x = r$ so that in this case $r = 10$ ft.

(*a*) Periodic time $= \dfrac{2\pi}{\omega} = \dfrac{2\pi}{1/\sqrt{5}} = 14{\cdot}05$ sec.

(*b*) Velocity 6 ft. from centre $= (1/\sqrt{5})\sqrt{(100 - 36)}$
$= 8/\sqrt{5} = 3{\cdot}578$ ft./sec.

(*c*) Acceleration 6 ft. from centre $= \omega^2 x = (1/5) \times 6$
$= 1{\cdot}2$ ft./sec.^{2}

Note. Since the acceleration in s.h.m. is proportional to the displacement and always directed towards a fixed point the differential equation of the motion is

$$\text{Acceleration} = d^2x/dt = -p^2x \text{ where } p \text{ is a constant}$$

or
$$d^2x/dt^2 + p^2\,x = 0 \tag{i}$$

Hence if we can show any motion to be represented by such an equation, we can immediately say that this motion is Simple Harmonic. For example in Problem 1 the equation of motion is obviously

$$d^2x/dt^2 + \omega^2x = 0.$$

The periodic time was given as $t = 2\pi/\omega$ so that for any s.h.m. as represented by (i)

$$t = 2\pi/p$$

3. A weight of 40 lb. attached to the lower end of a vertical spiral spring makes small vertical oscillations. If the stiffness of the spring is such that a weight of 100 lb. extends it 1 in., find the frequency of the vibrations. Neglect the weight of the spring.

SOLUTION. Imagine the weight to be displaced a distance x ft. vertically downwards from its equilibrium position. Then,

$$\text{Tension in spring} = \text{T} = 1\,200x \text{ lb.}$$

Making use of the relation,

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

to form the equation of motion

$$(\text{W}/g)\,.\,(d^2x/dt^2) + (\text{W}/g)g = \text{W} - \text{T}$$

or
$$d^2x/dt^2 + g\text{T}/\text{W} = 0$$

$\therefore$
$$d^2x/dt^2 + (g/\text{W})\,.\,1\,200x = 0.$$

This obviously represents a s.h.m. and we have

$$\text{Frequency of vibrations} = \text{number/sec.} = 1/t$$

$$= \frac{(g \times 1\,200)/\text{W}}{2\pi} = \frac{32{\cdot}2 \times 1\,200}{40 \times 2\pi}$$

$$= 153{\cdot}8 \text{ per sec.}$$

4. A particle moves with s.h.m.; if its velocities at distances of 4 and 6 ft. from the centre of its path are 3 and 1 ft./sec. respectively, find the periodic time and amplitude of the motion, and the acceleration in the extreme positions.

Ans. $t = 9{\cdot}931$ sec.
$r = 6{\cdot}204$ ft.
$f = 2{\cdot}481$ ft./sec.^{2}

5. A connecting-rod (Fig. 28) of weight W and radius of gyration k about its c.g. oscillates about an axis through its gudgeon-pin centre. Find the time of a small oscillation if the distance from the axis of suspension to the c.g. is h.

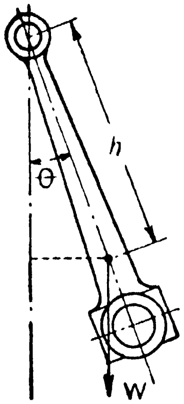

FIG. 28

SOLUTION. Taking moments about the centre of suspension we have by the principles of Section 3,

$$\text{Couple} = \text{Moment of inertia} \times \text{Angular acceleration}.$$

Hence, if the positive direction of θ, the angular displacement, is as shown in Fig. 28,

$$I_0 \times \ddot{\theta} = -Wh \,.\, \sin\theta$$

or $$(W/g)(k^2 + h^2)\,\ddot{\theta} = -Wh \,.\, \sin\theta.$$

Now for very small displacements we may put $\sin\theta = \theta$

$$\therefore \qquad [(k^2 + h^2)/g]\ddot{\theta} = -h\theta$$

or $$\ddot{\theta} + gh/(k^2 + h^2) \,.\, \theta = 0.$$

This is the equation of motion of the rod and represents a simple harmonic angular motion.
The time of a complete small oscillation is

$$t = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{\theta}{\ddot{\theta}}} = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}$$

6. A weight of 20 lb. attached to the free end of a horizontal cantilever, of length 12 in., performs small vertical oscillations. Neglecting the weight of the cantilever, find the frequency of the oscillations. Take Young's

modulus for cantilever material $= 30 \times 10^6$ lb./in.^{2} and the uniform section of the cantilever to be 1 in. wide by $\frac{3}{16}$ in. deep.

Ans. Frequency $= 3{\cdot}744$ per sec.

7. A uniform circular plate of radius R performs small free oscillations about a horizontal axis through a point on its circumference. Show that the periodic time is given by

$$t = 2\pi\sqrt{(3R/2g)}.$$

8. A homogeneous cone of base radius R and height h oscillates in a vertical plane about a horizontal axis through its apex. Prove that the time of a small oscillation is given by

$$t = 2\pi\sqrt{[(R^2 + 4h^2)/5gh]}.$$

9. The diagram (Fig. 29) shows the mechanism of a sleeve valve. If the crank makes 1 200 r.p.m., find the

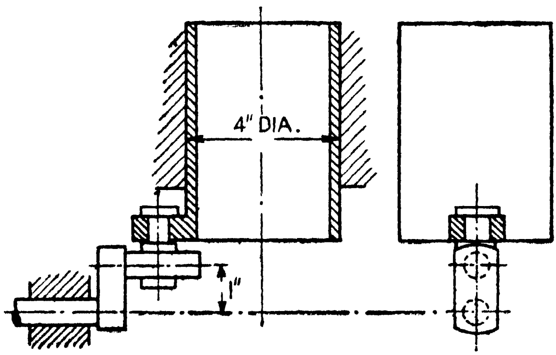

FIG. 29

vertical and angular velocities of the sleeve when the crank is in a position making an angle of 45° with the top dead centre position.

Ans. 7·4 ft./sec.
47·6 radn./sec.

10. A homogeneous hemisphere of radius a performs small free oscillations on a perfectly rough horizontal plane. Find the period of a complete oscillation.

SOLUTION. G (Fig. 30) is the centre of gravity of the hemisphere, and O, the point of contact, is evidently the instantaneous centre of rotation.

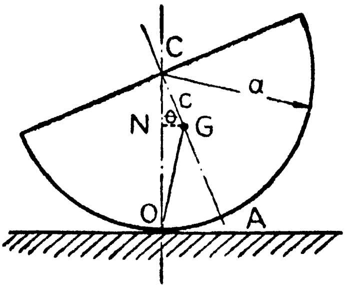

FIG. 30

Let $CG = c$ and weight of hemisphere $=$ W. Taking moments about O, the instantaneous centre,

$$(W/g)(k^2 + GO^2)(d^2\theta/dt^2) = -Wc \sin\theta$$

This is the exact equation of motion for any displacement θ. For very small oscillations we may take the *mean or undisturbed* position of the instantaneous centre, i.e. $GO = GA = (a - c)$ and also $\sin\theta = \theta$.

The equation of motion therefore becomes,

$$[k^2 + (a - c)^2](d^2\theta/dt^2) = -gc\,.\,\theta$$

or $\dfrac{d^2\theta}{dt^2} + \dfrac{gc}{k^2 + (a - c)^2}\,.\,\theta = 0$, which represents a s.h.m. Hence the period of a small oscillation is,

$$t = 2\pi\sqrt{\frac{k^2 + (a - c)^2}{gc}}$$

Here $\qquad k =$ radius of gyration about G.

Now from previous Problems (Section 2),

$$k^2 + c^2 = 2a^2/5, \text{ and } c = 3a/8$$

$$\therefore \quad k^2 + (a - c)^2 = 2a^2/5 + a^2 - 2a \times 3a/8$$
$$= 13a^2/20$$

$$\therefore \quad t = 2\pi\sqrt{\frac{13a^2/20}{g \times 3a/8}} = 2\pi\sqrt{\frac{26a}{15g}} \text{ sec.}$$

11. A homogeneous cylinder of radius a makes small oscillations inside a rough fixed cylinder of internal radius R. Find the time of a small oscillation.

Ans. $t = 2\pi\sqrt{(3R/2g)}$ sec.

12. If the plane in Problem 10 were perfectly smooth, show that the period of oscillation would be $2\pi\sqrt{(16a/15g)}$.

(*Hint:* The motion of G must now be vertical and that of O is horizontal. We therefore take moments about N in this case since it lies on the instantaneous axis.)

13. A horizontal shaft of length l and diameter d supported in bearings is fixed rigidly at one end and carries at the other a heavy fly-wheel of weight W. If t is the time of a small torsional oscillation of the fly-wheel show that, neglecting the weight of the shaft, the radius of gyration of the fly-wheel is given by $k^2 = gCd^4t^2/128\pi Wl$, where C is the modulus of rigidity of the material of the shaft.

SOLUTION. Suppose the free end to have twisted through a small angle θ due to some impulse. Then the theory of torsion of circular shafts gives,

$$\mathrm{T/I} = \mathrm{C}\theta/l \text{ where } \mathrm{T} = \text{applied torque}$$

$$\mathrm{I} = \text{second moment of shaft about axis}$$

$$\mathrm{C} = \text{modulus of rigidity.}$$

$$\therefore \quad \text{Restoring couple} = \mathrm{T} = \mathrm{CI}\theta/l$$

$$= \mathrm{C}\pi d^4\theta/32l.$$

Taking moments about the axis at the free end,

$$\frac{\mathrm{W}}{g} \cdot k^2 \cdot \frac{d^2\theta}{dt^2} = -\frac{\mathrm{C}}{l} \cdot \frac{\pi d^4}{32} \cdot \theta$$

or
$$\frac{d^2\theta}{dt^2} + \frac{g}{\mathrm{W}k^2} \cdot \frac{\mathrm{C}}{l} \cdot \frac{\pi d^4}{32} \cdot \theta = 0,$$

which represents a simple harmonic motion.

$$\therefore \qquad t = 2\pi\sqrt{\frac{\mathrm{W}k^2 \cdot l \cdot 32}{g\mathrm{C}\pi d^4}}$$

or
$$t^2 = \frac{4\pi^2 \cdot 32\mathrm{W}k^2 \cdot l}{g\mathrm{C}\pi d^4}$$

i.e.
$$k^2 = \frac{g\mathrm{C}d^4 \cdot t^2}{128\pi\mathrm{W}l}.$$

14. A body of weight 80·5 lb. has a simple harmonic motion under the action of a force of 31·25 lb. per ft. displacement. Find the period of a vibration. If the body be retarded by a frictional force proportional to the speed and equal to 3·75 lb. per ft./sec., find the new period.

(*W.S.S.*)

Ans. 1·77 sec.
1·82 sec.

15. A plumb-line, consisting of a small mass suspended from a string of length l, is attached to the roof of a railway carriage. With the carriage travelling along a straight track at a uniform speed v, the plumb-line is vertical and stationary relative to the carriage.

If the carriage suddenly enters a curve of radius r, show that the plumb-line will start to oscillate, the deflection from the vertical being given by

$$\theta = (v^2/gr)\{1 - \cos [t \,.\, \sqrt{(g/l)}]\}$$

(*Mech. Sc. Tripos Cam.*)

16. In order to find the moment of inertia of an accurately balanced turbine disc of mass M, keyed to a shaft of radius r ft., the disc is placed with the shaft horizontal and resting on level surfaces on either side, and a small mass m is fixed to the wheel at a distance h ft. from the axis of the shaft. If T sec. is the time of a complete small oscillation, prove that the moment of inertia of the wheel is given by

$$\mathrm{I} = m[gh\mathrm{T}^2/4\pi^2 - (h - r)^2] - \mathrm{M}r^2.$$

(*L.U.A.*)

5. GEOMETRICAL PROPERTIES—VELOCITY, ACCELERATION AND EQUILIBRIUM OF MECHANISMS

1. In the four bar mechanism (Fig. 31) find the limiting positions of B and the corresponding angular positions of EA for one revolution of this latter. What position of EA corresponds to the mid-point of the path of B?

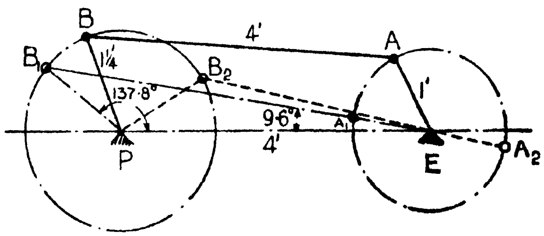

FIG. 31

SOLUTION. An examination of the mechanism will show that the limiting positions of B occur when E, A and B lie in a straight line. It follows that B_1 is determined by $B_1\widehat{D}E$ of the triangle EB_1D. By the cosine rule,

$$\cos B_1\widehat{D}E = \frac{(B_1D)^2 + (DE)^2 - (B_1E)^2}{2(B_1D)(DE)}$$

$$= \frac{(1{\cdot}25)^2 + (4)^2 - (5)^2}{2 \times 1{\cdot}25 \times 4} = -\,0{\cdot}7438$$

$$\therefore \qquad B_1\widehat{D}E = 137{\cdot}8^\circ.$$

The corresponding position of A is determined by $D\widehat{E}B_1$. This may be found by the sine rule, thus,

$$\frac{B_1E}{\sin B_1\widehat{D}E} = \frac{B_1D}{\sin D\widehat{E}B_1}$$

$$\therefore \qquad \sin D\widehat{E}B_1 = \frac{1{\cdot}25 \times 0{\cdot}67}{5} = 0{\cdot}1675$$

$$\therefore \qquad D\widehat{E}B_1 = 9{\cdot}6^\circ.$$

The other limiting position, namely B_2, is found in a similar manner, and hence the mid-point of the path of B may be determined and the corresponding position of A follows immediately.

2. In the mechanism shown in Fig. 32, the ends A and B of the link AB are constrained to move in the slots OY

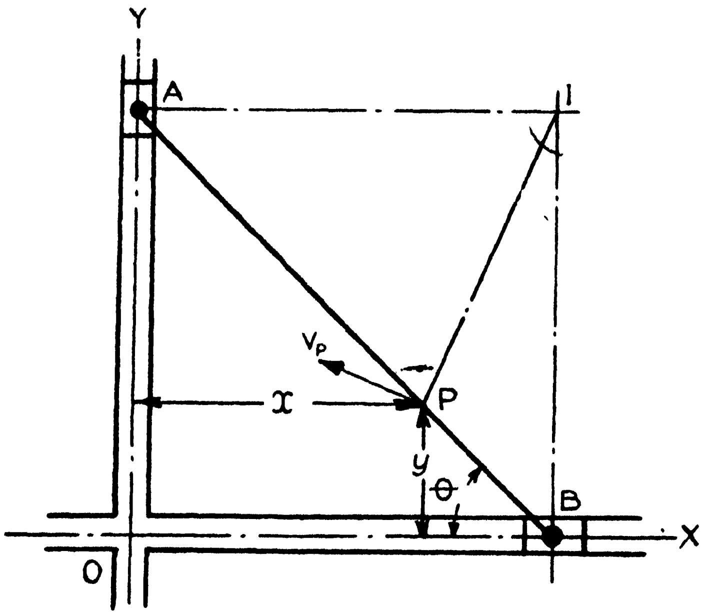

FIG. 32

and OX, which are fixed at right angles. Show that the path traced by the point P is an ellipse of which AP and BP are the semi-major and semi-minor axes respectively.

SOLUTION. Taking OX and OY as the co-ordinate axes, and x and y as the abscissa and ordinate of the point P in the position shown,

Let $AP = a$, and $BP = b$.

We have from the geometry of the mechanism,

$$x = a \cos \theta,\ y = b \sin \theta$$

$$\therefore \qquad x/a = \cos \theta,\ y/b = \sin \theta.$$

Squaring and adding we get,

$$x^2/a^2 + y^2/b^2 = (\sin^2 \theta + \cos^2 \theta) = 1.$$

Hence the path traced by the point P as θ changes is given by the equation,

$$x^2/a^2 + y^2/b^2 = 1.$$

Now this equation represents an ellipse having principal axes OX and OY and semi-major and semi-minor axes a and b respectively. This mechanism is known as the *elliptic trammel*, and any point P on the link AB describes an ellipse. When P is in the middle of AB the ellipse is the circle of radius $\frac{1}{2}AB$.

3. If the slider B of the previous question moves along OX towards O with a velocity of 20 ft./sec., what is the velocity of the point P, and the velocity of the point A for the position shown? Take AB to represent 3 ft. 3 in. to scale.

SOLUTION. Two simple methods are available—

(i) Making use of the known properties of the instantaneous centre.

(ii) Drawing the velocity diagram for the mechanism.

Both methods are given below.

(i) Erect the perpendiculars IA and IB to OX and OY respectively. Then I is the instantaneous centre of rotation of the link AB. Hence, by the known properties of the instantaneous centre.

$$IA/IB = V_A/V_B \text{ and } IP/IB = V_P/V_B$$

and the velocity of any point on the link AB is normal in direction to the line joining that point to I.

For the position shown,

$IA = 2{\cdot}3$ ft., $IB = 2{\cdot}3$ ft., $IP = 1{\cdot}7$ ft., and $V_B =$ 20 ft./sec.

Therefore,

$$V_A = \frac{IA}{IB} \times V_B = \frac{2{\cdot}3}{2{\cdot}3} \times 20 = 20 \text{ ft./sec. along } OY$$

$$V_P = \frac{IP}{IB} \times V_B = \frac{1{\cdot}7}{2{\cdot}3} \times 20 = 14{\cdot}8 \text{ ft./sec.}$$ (as indicated in Fig. 32).

(ii) Select a suitable scale of velocity (as large as possible), and taking any point e, draw eb (Fig. 33) in the direction of the motion of B, to represent 20 ft./sec. Draw a line through e parallel to OY and through b draw a line at right angles to the link AB to cut the line parallel to OY in a. Divide ba in p so that

$$bp/ap = BP/AP.$$

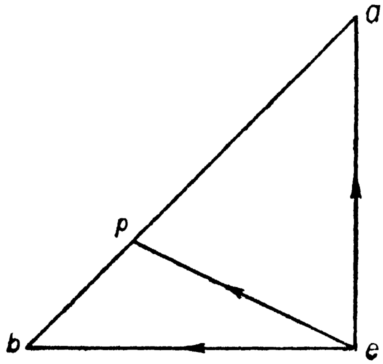

FIG. 33

Draw ep.

Since eb = velocity of B in magnitude and direction, then ea = velocity of A in magnitude and direction and ep = velocity of P in magnitude and direction.

Note also that ba represents the velocity of A relative to B, and hence the angular velocity of the link AB is ba/AB radn./sec.

Scaling off the required velocities we get

$$V_A = ea = 20 \text{ ft./sec.}$$

$$V_P = ep = 14{\cdot}8 \text{ ft./sec. in directions shown.}$$

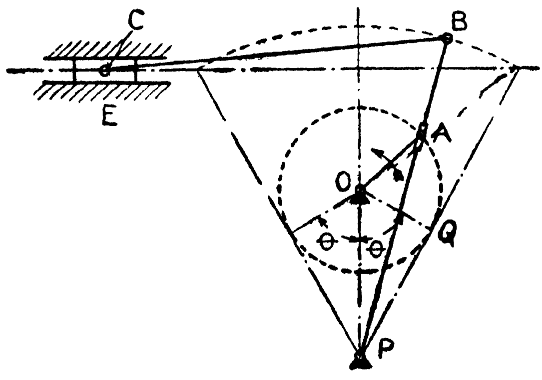

FIG. 34

4. The diagram (Fig. 34) shows a crank and slotted lever quick-return motion. If the time of cutting is 3 sec., what is the time of return and the uniform speed of the crank OA?
$OA = 6$ in.; $OP = 12$ in.; $PB = 24$ in.; $BC = 25{\cdot}5$ in.

SOLUTION.

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{(360 - 2\theta)}{2\theta}$$

Now $\cos \theta = OQ/OP = 6/12 = 0{\cdot}5 \quad \therefore \theta = 60°.$

Hence

$$\text{Time of return} = \frac{2\theta}{(360 - 2\theta)} \times 3 \text{ sec.}$$

$$= \frac{120 \times 3}{(360 - 120)} = 1{\cdot}5 \text{ sec.}$$

It follows that the crank OA makes one complete revolution in $(3 + 1{\cdot}5) = 4{\cdot}5$ sec.

$\therefore$ Speed of crank $OA = 60/4{\cdot}5 = 13\frac{1}{3}$ r.p.m.

5. Find the velocity of the slider C and the angular velocity of the link BC in the previous example, when the crank OA is in the position shown in Fig. 34.

SOLUTION. The velocity diagram method will be used to illustrate the most general way of solving such problems.

$$\text{Angular velocity of crank } OA = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60 \times 3}$$

$$= \frac{4\pi}{9} \text{ radn./sec.}$$

$\therefore$ Velocity of point A on crank $OA = (4\pi/9) \times OA$

$$= (4\pi/9) \times \tfrac{1}{2} = 0{\cdot}7 \text{ ft./sec.}$$

This velocity is directed tangentially to the circle AQ in an anti-clockwise sense.

Select a scale of velocity (as large as possible).

(i) Draw ea (Fig. 35) at right angles to OA = velocity of A, 0·7 ft./sec. The point e is called the *earth point* and ea is actually the velocity of A relative to the earth.

(ii) Through e draw a line normal to PB, of indefinite length.

(iii) Through a draw a line parallel to PB, cutting (ii) in a_1. Then ea_1 = velocity of point on PB which coincides with A. And a_1a = velocity of slider along PB.

(iv) Produce ea_1 to b, so that $ea_1/eb = PA/PB$. Then eb = velocity of point B.

(v) Through e draw a line parallel to line of action of slider at C.

(vi) Through b draw a line normal to link BC cutting (v) in c.

Then since bc represents the velocity of C relative to B, ec gives the velocity of the slider at C.

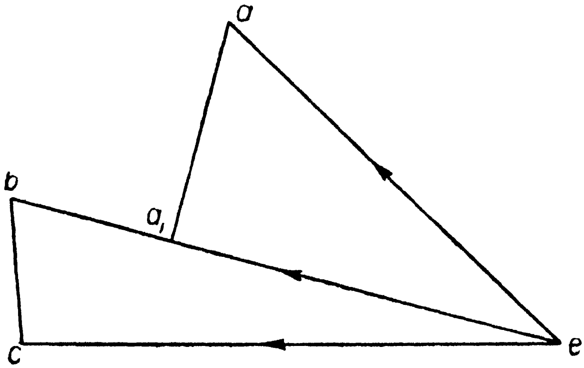

FIG. 35

Velocity of slider at C = ec in direction e to c
= 0·83 ft./sec.

Angular velocity of link BC

$$= \frac{\text{Velocity of } C \text{ relative to } B}{\text{Length of } CB}$$

$$= bc/BC = (0{\cdot}215/25{\cdot}5) \times 12$$

$$= 0{\cdot}1 \text{ radn./sec.}$$

6. What is the acceleration of the slider C and that of the slider at A along PB, in Problem 4, for the given position?

SOLUTION. A graphical method is again utilized and is recommended for future problems. It should be noted that in general the acceleration of a point in a link is made up of two components considered relative to the centre of rotation of the link—

(*a*) A radial acceleration directed along the link of magnitude $\omega^2 r$, where ω is the angular velocity of the link and r is the distance of the point from the centre of rotation of the link.

(*b*) An acceleration normal to the link which is usually unknown but has the form $\dot{\omega}r$, where $\dot{\omega}$ is the angular acceleration of the link.

Tabulating the known radial accelerations of the various links we make use of the velocity diagram of Problem 5.

ω for OA = 1·4 radn./sec. $\therefore\ \omega^2 . OA$. = 0·97 ft./sec.^{2}

ω for PB = 0·437 radn./sec. $\therefore\ \omega^2 . PB$. = 0·386 ft./sec.^{2}

ω for BC = 0·10 radn./sec $\therefore\ \omega^2 . BC$. = 0·021 ft./sec.^{2}

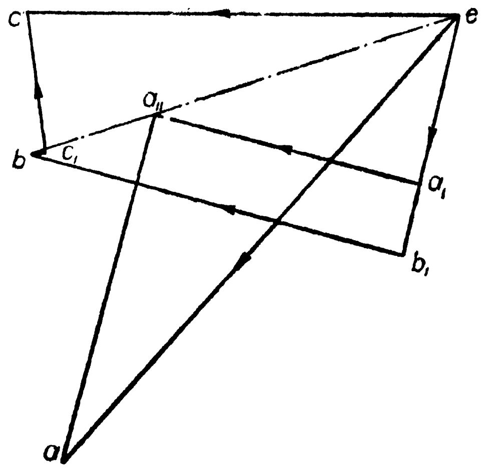

FIG. 36

Selecting any earth point *e*, as in the velocity diagram, we proceed as follows (Fig. 36).

(i) Draw *ea* in the direction *AO* to represent the radial acceleration of *A*. Since *OA* has a uniform angular velocity *A* has no acceleration normal to *OA*.

(ii) Draw eb_1 in the direction BP = radial acceleration of *B*, and erect a line perpendicular to eb_1 at b_1.

(iii) Divide eb_1 at a_1 so that $ea_1/eb_1 = PA/PB$, and erect a perpendicular to ea_1 at a_1.

(iv) Draw aa_{11} parallel to *BP*.

(v) Join ea_{11} and produce to cut normal at b_1 in *b*. Then

a_1a_{11} = acceleration of point *A* on *PB*, normal to *PB*.

b_1b = acceleration of point *B* on *PB*, normal to *PB*.

aa_{11} = acceleration of slider at *A* along *PB*.

(vi) Draw bc_1 = radial acceleration of C relative to B and erect a perpendicular to bc_1 at c_1.

(vii) Draw a line through e parallel to the line of action of C to cut normal at c_1, in c.

Then,

c_1c = acceleration of C normal to BC and relative to B.

ec = acceleration of slider C along its line of action.

Scaling off the required accelerations from the diagram,

Acceleration of slider C = 0·725 ft./sec.^{2} in direction e to c.

Acceleration of slider A along PB = 0·575 ft./sec.^{2} in direction a_{11} to a.

Since the slider at A is actually moving towards B this means a retardation in this latter direction.

7. If a force of 100 lb. acts at A at right angles to OA, and in an anti-clockwise sense, what force at C along the line of action of C will keep the mechanism of Problem 4 at rest?

SOLUTION. The force at A must be resolved into two components which must pass through P and B respectively. Obviously this could be done in an infinite number of ways, but for our purpose the most convenient way is to select the component through B so that it acts along BC.

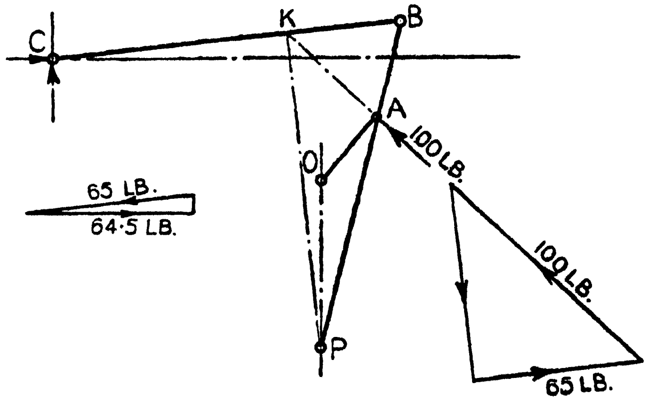

FIG. 37

Thus, as in Fig. 37, produce the direction of the force at A to cut BC in K, and join K to P. Then BK and PK are

the directions of the components through B and P respectively. Resolving the force at A in these directions by a triangle of forces, we find a force of 65 lb. acting along BC.

For equilibrium this must be balanced by a force along the line of action of C, together with a force perpendicular to this direction, both acting at C.

Drawing a triangle of forces for this point we obtain the required force.

Force at C, in line of action of C = 64·5 lb.

8. The diagram (Fig. 38) illustrates the mechanism of a radial engine. The piston A_0 is operated by the master

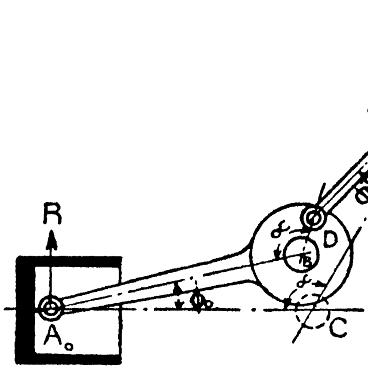

FIG. 38

connecting-rod BA_0 and A_1 is operated by the articulated connecting-rod DA_1 hinged to the "big-end" of the master rod.

If the crank CB is held fixed and the piston A_1 is subjected to a thrust P as shown, show that the force R acting on A_0 transversely to CB necessary to maintain equilibrium in the master connecting-rod is given by

$$R = P \,.\, (c/l)(\tan \phi_0 + \tan \phi_1)$$

where $l = A_0B$, $c = BD$ and ϕ_0, ϕ_1 are the obliquities of the connecting-rods. (*Mech. Sc. Tripos Cam.*)

9. Two equal links, AB and BC, 5 in. long are hinged together at B. The end A is pivoted to a fixed frame that carries also bearings for a crankshaft. The axis O of this shaft is 3 in. from A, and the end C of the second link is pivoted on the crank at radius OC = 1 in. Find the length

of the arc traversed by B and, also, its speed when equidistant from the two ends of this arc. The crankshaft rotates at 120 r.p.m. (*I.M.E.*)

Ans. 3·559 in.
1·44 ft./sec. *or* 2·62 ft./sec.

10. A plane linkwork $ABCDE$ comprises five members, including the horizontal base AE. The connexion C lies above B and D, which lie above the base. The lengths of the members are as follow: $AE = 8$ in.; $AB = 4$ in.; $BC = 4$ in.; $CD = 3$ in.; DE = 5 in. Draw the configuration when B and D are equally distant from the base AE, and 6 in. apart from one another. If C is moving horizontally at 12 in./sec., find the rate at which the distance BD is changing. (*I.M.E.*)

Ans. 2·17 ft./sec.

11. Fig. 39 shows Peaucellier's linkwork. Prove that when P describes a circle about B as centre, Q also describes a circle. Find the position of this circle and its radius.

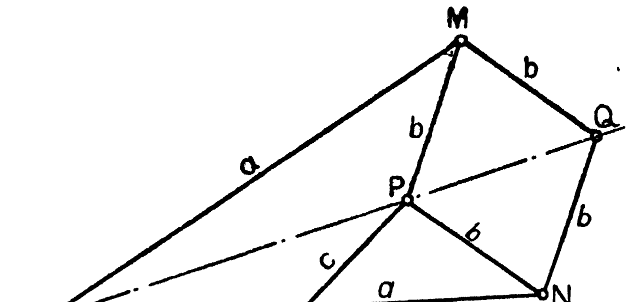

FIG. 39

12. Taking the lengths of the links in Problem 11 to be, $a = 12$ in.; $b = 4$ in.; $c = 4$ in.; $d = 6$ in.; draw velocity and acceleration diagrams for the position shown when the crank PB is rotating with a uniform speed of 200 r.p.m. Hence find the velocity and acceleration of the point Q.

Ans. 10·4 ft./sec.; 103 ft./sec.^{2}

13. If a torque of 30 lb. ft. is applied to the crank PB of Fig. 39, find the horizontal force at M which will keep the mechanism in equilibrium.

Ans. 223 lb.

14. In the mechanism shown in Fig. 40, A is constrained to slide along XX while EB swings about E. Find the

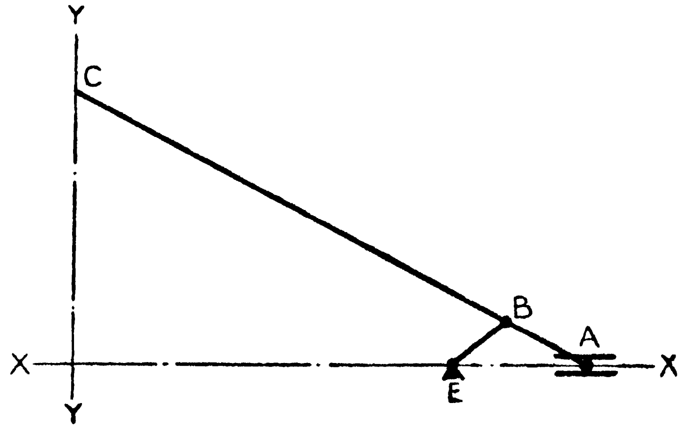

FIG. 40

relation which must exist between the lengths AB, BC and EB, so that C will move along a straight line YY, normal to XX, for a small angular displacement of AB on each side of XX.

$$\textit{Ans.}\ EB = \frac{AB^2}{BC}.$$

15. Fig. 41 shows diagrammatically the Andreau differential stroke engine. The two cranks are geared together with a velocity ratio of 2 : 1. Adopting a scale of $CD = 1{\cdot}5$ ft. for the diagram, draw the velocity and acceleration diagrams for the mechanism and hence determine the velocity and acceleration of the connecting-rod "big-end."

Ans. Velocity $= 0{\cdot}188\omega$.
Acceleration $= 0{\cdot}328\omega^2$ where $\omega =$ angular velocity of smaller crank.

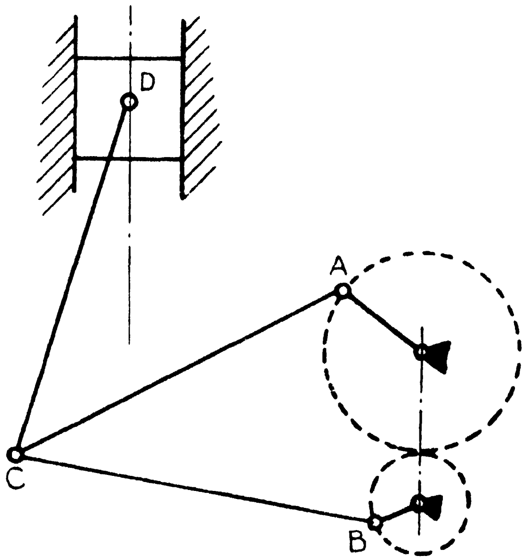

FIG. 41

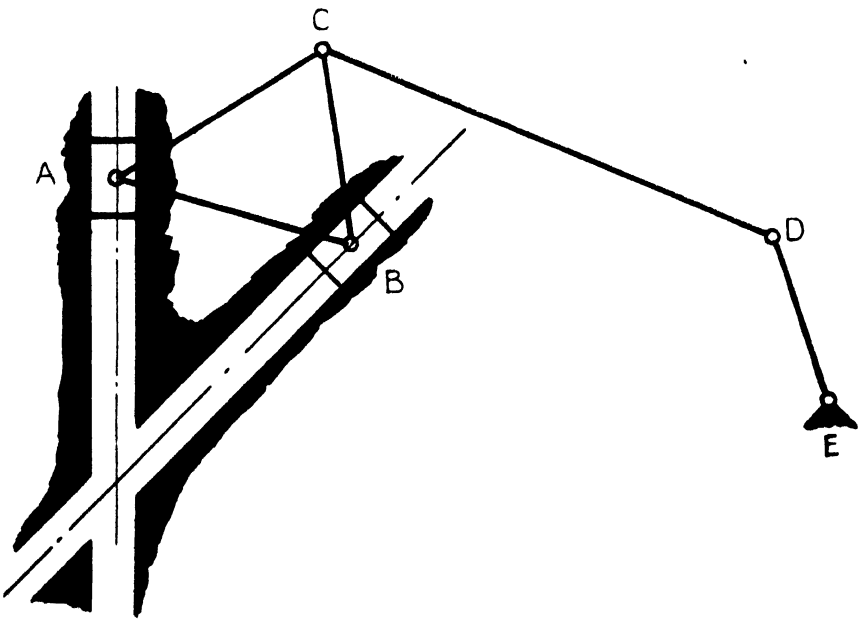

FIG. 42

16. What torque acting on the low speed crank in Problem 15 will keep the mechanism at rest when a force of 250 lb. acts on the piston along the line of stroke?

Ans. 32·5 lb. ft.

17. Find the path traced by the point C of the mechanism shown in Fig. 42, and indicate its axes of symmetry.

18. If the crank ED of Fig. 42 has a uniform angular velocity of 10 radn./sec. in the given position, find the velocity of the slider A and the angular velocity of the element ABC.

Take $AB = 1$ ft. in the diagram.

(*Hint:* Assume any velocity for B and, using this value, draw the velocity diagram. The known velocity of D may then be used to correct the scale of this diagram.)

Ans. Velocity of A = 6·5 ft./sec.
Angular velocity of ABC = 9·43 radn./sec.

19. What force acting vertically at D, in Fig. 42, will keep the mechanism at rest when a force of 200 lb. acts at B along its line of action?

Ans. 520 lb.

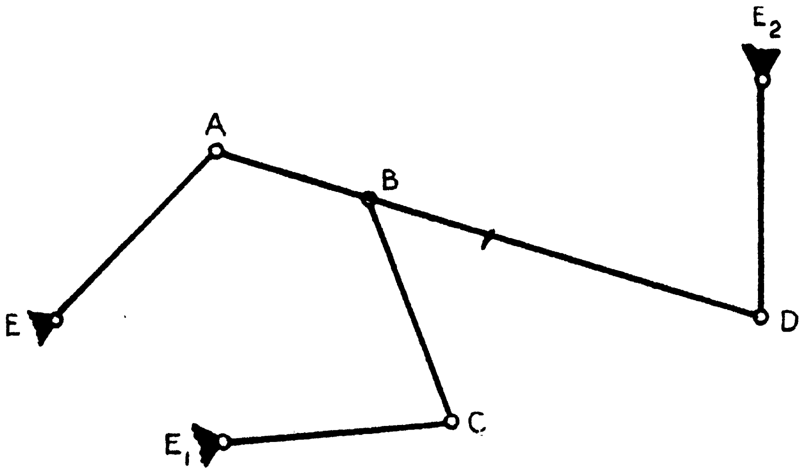

FIG. 43

20. Given that the angular velocity and angular acceleration of the link E_1C (Fig. 43) are 20 radn./sec. and 2·5 radn./sec.^{2} respectively for the given position, draw velocity

and acceleration diagrams for the mechanism. Take scale of figure 1 in. = 1 ft.

21. Two shafts are coupled together by a Hooke's joint, the driving shaft rotating uniformly at 600 r.p.m. Find the greatest permissible angle between the centre-lines of the shafts, if the maximum speed of the follower shaft is 630 r.p.m. Prove your reasoning. What is then the minimum speed of this shaft? State the conditions under which two shafts connected by a double Hooke's joint have the same angular velocities. (*L.U.A.*)

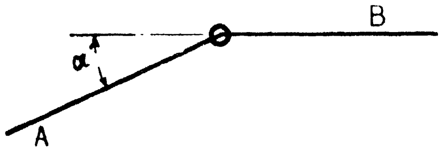

FIG. 44

SOLUTION. Let α (Fig. 44) be the angle between the axes of the two shafts. Then if

$$\omega = \text{angular velocity of shaft } B$$

and

$$\omega_1 = \text{angular velocity of shaft } A$$

it may be shown (the proof is left to the student) that

$$\omega/\omega_1 = (1 - \sin^2\alpha \,.\, \cos^2\theta)/\cos\alpha$$

where θ is the angle turned through by the shaft A from the position shown.

ω is a maximum when $\theta = \pi/2$ and a minimum when $\theta = 0$.

$$\therefore \quad 630/600 = (1 - \sin^2\alpha \times 0)/\cos\alpha = 1/\cos\alpha$$

$$\therefore \quad \cos\alpha = 0{\cdot}9526$$

and $\alpha = 17{\cdot}7^\circ$ is the required angle between the shafts. The minimum speed of the follower is obtained when $\theta = 0$.

$$\therefore \quad \omega_{\text{(min.)}} = 600\,(1 - \sin^2\alpha)/\cos\alpha$$

$$= 600\cos\alpha = 600 \times 0{\cdot}9526$$

$$= 571{\cdot}56 \text{ r.p.m.}$$

22. A modification of Oldham's coupling is shown diagrammatically in Fig. 45. A and B are two parallel shafts, arranged with their axes at distance a apart and carrying perpendicular rods AC and BD respectively. A connecting-rod EF, of length $2a$, slides with its end E on AC and its

end F on BD, the angle AEF being fixed at α. The slides are such that AC and BD are always perpendicular to one another. P is the mid-point of EF. Show that, when AC

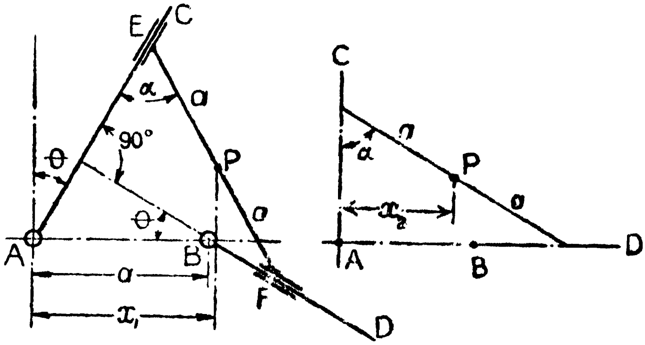

FIG. 45

has turned through an angle θ as shown, the velocity of P in the direction AB is

$$a\omega[\sin 2\theta + \cos(\theta + \alpha)]$$

where ω is the angular velocity of the shafts.

(*Mech. Sc. Tripos Cam.*)

SOLUTION. When AC was vertical the horizontal distance of P from A was $x_2 = a \sin \alpha$.

In the position shown the horizontal distance of P from A is

$$\begin{aligned} x_1 &= AE \sin \theta + EP \sin (\alpha - \theta) \\ &= (2a \cos \alpha + a \sin \theta) \sin \theta + a \sin (\alpha - \theta) \\ &= a(2 \cos \alpha \sin \theta + \sin^2 \theta + \sin \alpha \cos \theta - \cos \alpha \sin \theta). \end{aligned}$$

Hence the horizontal displacement of P for angular displacement θ is

$$x = (x_1 - x_2)$$

$$= a(2 \cos \alpha \sin \theta + \sin^2 \theta + \sin \alpha \cos \theta - \cos \alpha \sin \theta - \sin \alpha).$$

The horizontal velocity of P is given by dx/dt.

$$\begin{aligned} dx/dt &= a(2 \cos \alpha \cos \theta + 2 \sin \theta \cos \theta - \sin \alpha \sin \theta \\ &\qquad - \cos \alpha \cos \theta)\, d\theta/dt. \\ &= a(d\theta/dt)[\sin 2\theta + \cos (\theta + \alpha)] \\ &= a \,.\, \omega[\sin 2\theta + \cos(\theta + \alpha)], \text{ since } (d\theta/dt) = \omega. \end{aligned}$$

23. The diagram (Fig. 46) shows a transverse section through the rotating parts of a pump designed for dealing with viscous fluids.

PPP are three pistons fixed by rigid rods to a boss *B* which is keyed to a driving shaft.

CCC are rectangular blocks bored to receive the pistons. The ring *R* is bolted to two end discs and the blocks *CCC* are in sliding contact with these discs so that they can move freely to and fro in the cavity formed between the ring and the two discs, in a direction at right angles to the axes of the respective pistons. The discs are carried in bearings with axes parallel to the axis of the shaft, but eccentric to it. (The diagram is drawn with the axis of the shaft above that of the disc bearings.) The casing of the pump is not shown. When power is put into the shaft the ring and discs are driven by the pistons through the medium of the blocks *CCC*. Given the speed of the shaft, determine the speed of the ring and derive expressions for the velocity and acceleration of the centre of gravity of one of the blocks, assuming that this point lies on the centre line of the corresponding piston. (*W.S.S.*)

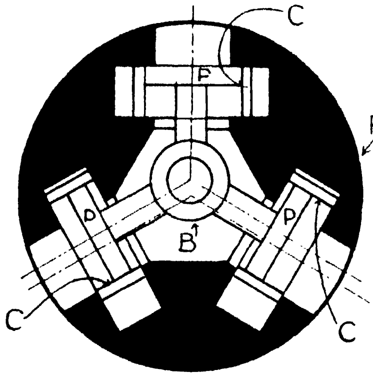

Fig. 46

24. Fig. 47 represents a right- and left-hand screw steering gear for operating a ship's rudder. *XY* is the centre line of the mechanism. The axis of the rudder post is shown at *O* and the rudder is moved by a rigid crosshead, the arms of which are shown as *OA* and *OB*. Connecting-rods *AC* and *BD* connect the crosshead with two sliders which move along guide-rods parallel to *XY*, being propelled by the right- and left-hand screw *S* which engages with nuts N_1 and N_2 integral with the two sliders, respectively. According to the direction in which the screw is turned the nuts are drawn together or forced apart, but a small endwise motion of the screw is also entailed.

Draw, with sufficient accuracy to indicate its general character, a curve showing the ratio between the linear velocity of the slider C and that of the crank-pin A, for

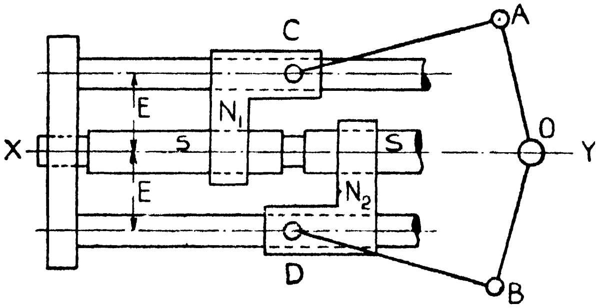

Fig. 47

values of the angle AOX ranging from 45° to 135°. Take $OA = OB = 18$ in., $AC = BD = 40$ in., and distance $E = 12$ in.

From the form of this curve deduce that a considerable shift of the screw must be allowed for as the crosshead turns, if the angle AOB is 180°, but that, by slightly reducing this angle in the direction indicated in the figure, the amount of this shift may be greatly reduced. (*W.S.S.*)

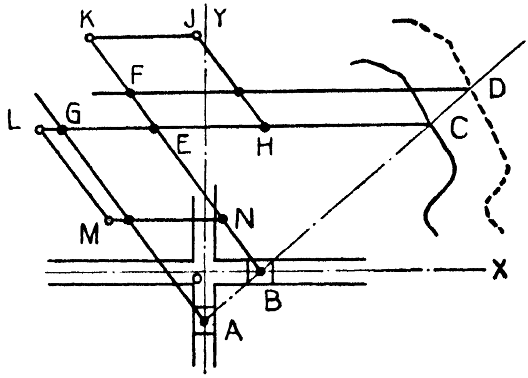

Fig. 48

25. Fig. 48 shows a mechanism known as Coté's proportional compasses. The links are all freely jointed and the eight sides of the parallelograms $EHJK$ and $ENML$ are equal. The points A and B are constrained to move along

OY and OX respectively, and the point C is moved over a given outline. Show that the point D will describe a second figure, resembling the first, in which the corresponding abscissæ will be a times greater and the corresponding ordinates b times greater than those of the first figure, provided the bars FD and GA are clamped so that

$$EF/EB = (b - 1) \text{ and } EG/EC = (b - a)/(a - 1)$$

26. A coplanar straight-line mechanism has two links AQ and PQ, equal in length, PQ being the fixed link of the mechanism. Another point B of the mechanism is constrained to move in such a way that the points A, P and B are always collinear and the product $AP \cdot PB =$ constant. Show that B describes a straight line perpendicular to PQ.

(*Mech. Sc. Tripos Cam.*)

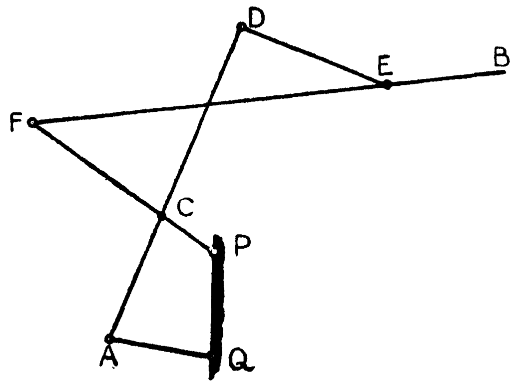

Fig. 49

27. In the mechanism shown in Fig. 49, if the links ACD, BEF, and PCF are continuous, $PQ = AQ$; $DE = FC$; $CD = EF$; $EB = AC$ and $PC/CF = AC/CD$; show that the point B describes a straight line perpendicular to PQ. (*Mech. Sc. Tripos Cam.*)

6. FRICTION IN MACHINES—LUBRICATION

1. A sledge whose weight is 30 lb., is pulled up a slope of 1 in 8 ($\sin^{-1} \frac{1}{8}$) by a rope which is inclined at an angle of 20° upwards to the surface of the slope. The coefficient of friction between the sledge and the slope is 0·05. Find the pull on the rope if the motion is uniform. (*W.S.*)

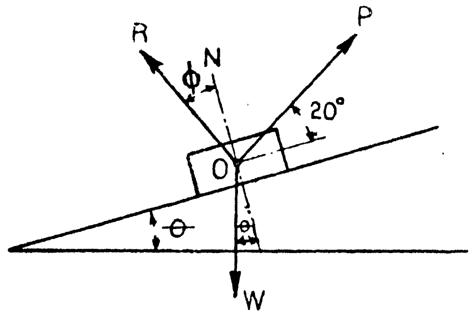

FIG. 50

SOLUTION. For uniform motion the three forces W, P, and R (Fig. 50) must be in equilibrium. If the motion is up the plane, the reaction R will make an angle ϕ with the normal to the plane *ON*, and to the left as shown.

$$\text{Coefficient of friction} = \tan\phi = 0{\cdot}05.$$

By Lami's theorem we have,

$$P/\sin[180° - (\theta + \phi)] = W/\sin[90° + (\phi - 20°)]$$

or
$$P = \frac{\sin(\theta + \phi)}{\cos(\phi - 20°)} \cdot W$$

$$= \frac{(\sin\theta\cos\phi + \cos\theta\sin\phi)}{(\cos\phi\cos 20° + \sin\phi\sin 20°)} \cdot W$$

$$\sin\theta = \tfrac{1}{8} \quad \therefore \cos\theta = (\sqrt{63})/8$$

$$\tan\phi = 1/20 \; \therefore \sin\phi = 1/\sqrt{401} \text{ and } \cos\phi = 20/\sqrt{401}$$

$$\sin 20° = 0{\cdot}342;\ \cos 20° = 0{\cdot}9397.$$

$$\therefore P = \frac{\frac{1}{8} \times \frac{20}{\sqrt{401}} + \frac{\sqrt{63}}{8} \times \frac{1}{\sqrt{401}}}{\frac{20}{\sqrt{401}} \times 0{\cdot}9397 + \frac{1}{\sqrt{401}} \times 0{\cdot}342} \times 30 = 5{\cdot}47 \text{ lb.}$$

2. For a square-threaded screw-jack with helical threads show that the maximum mechanical efficiency is given by $(1 - \sin\phi)/(1 + \sin\phi)$ where ϕ is the angle of friction, and that this occurs when the angle of the helix is $(\pi/4 - \phi/2)$ to the horizontal.

SOLUTION. The motion of a nut on a screw is analogous to motion on an inclined plane, the inclination of which is the angle of the helix. In the analogy (Fig. 51) the force P to produce uniform motion acts horizontally at the end of an arm of length l. Let the mean radius of the screw and nut be r.

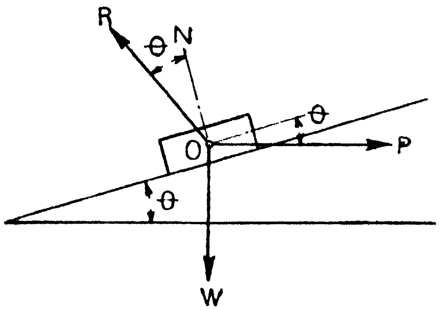

FIG. 51

Then by the method of Problem 1.

$$P = \frac{\sin(\theta + \phi)}{\cos(\theta + \phi)} \cdot W = W \cdot \tan(\theta + \phi) \text{ at radius } r.$$

The mechanical efficiency is given by

$$\eta = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}}$$

Mechanical advantage $= W/P'$ where P' corresponds to a radius l.

Now $\quad P' = Pr/l = W \cdot \tan(\theta + \phi) \cdot (r/l)$

$\therefore \quad$ Mechanical advantage $= l/r \tan(\theta + \phi)$.

$$\text{Velocity ratio} = \frac{\text{Distance moved by } P'}{\text{Distance moved by } W} \text{ in same time interval.}$$

$$= 2\pi l/(\text{pitch of screw}) = 2\pi l/2\pi r \tan\theta$$

$$= l/r \tan\theta.$$

Hence $\quad \eta = \dfrac{l/r \tan(\theta + \phi)}{l/r \tan\theta} = \dfrac{\tan\theta}{\tan(\theta + \phi)}.$

the resultant horizontal force each exerts on the drum is P. If the wear in the shoes at any point is proportional to the normal pressure at that point, show that after the shoes

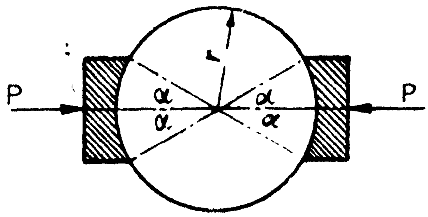

FIG. 57

have become "worn-in" the resistance to motion they provide is a couple of magnitude

$$[8 \sin \alpha/(2\alpha + \sin 2\alpha)] \cdot Pr\mu$$

where μ is the coefficient of friction.

(*Mech. Sc. Tripos Cam.*)

16. The diagram (Fig. 58) shows a form of steering gear known as Rapson's slide. Taking the angle of friction

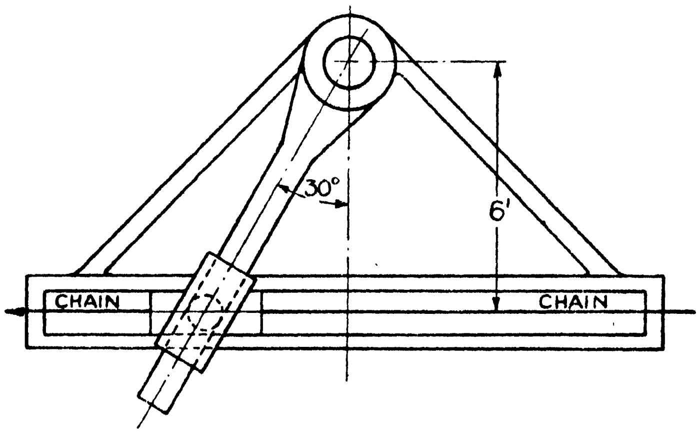

FIG. 58

between the two sliding pieces and their guides to be 5°, find the pull which must be applied to the chain to move the tiller to the left for the position shown. The couple to be overcome is 50 ton ft. Neglect friction at turning pairs.

Ans. 6·86 tons.

17. Particulars of a friction hoist which is designed to raise a load of $\frac{1}{2}$ ton are given in Fig. 59.

The friction wheel A is fixed to and concentric with a winding barrel B, and both are free to move on a shaft, the ends of which are "turned down" eccentrically with the main axis and are supported in fixed bearings. A lever with a weight W is keyed to one of the reduced ends of the shaft; raising this lever by means of a rope R liberates the wheel

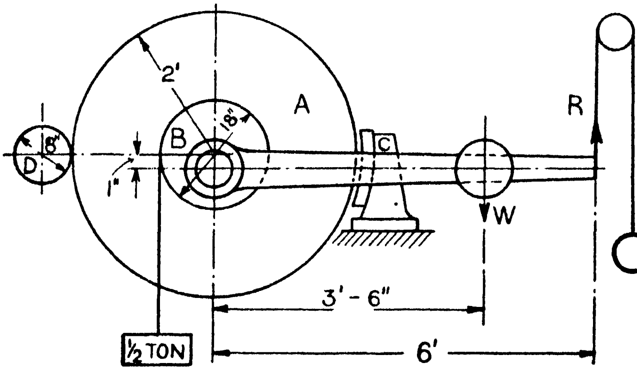

FIG. 59

A from the brake-block C and forces it against the driving pinion D. Calculate the weight W which is just sufficient to sustain the $\frac{1}{2}$ ton load, also the pull on the rope which is just sufficient to raise the $\frac{1}{2}$ ton load at an acceleration of 1 ft./sec.^{2} The lever weighs 20 lb. and its c.g. is 21 in. from the centre of the reduced end of the shaft. The moment of inertia of the wheel A and the winding barrel is 1 000 lb. ft.^{2} units. The coefficient of friction between the wheel A and the pinion D is 0·18, and between the wheel A and the brake-block is 0·33. (*L.U.A.*)

Ans. (i) 20·3 lb.
(ii) 70·36 lb.

18. Distinguish between boundary and film lubrication. Discuss carefully the conditions under which lubrication takes place between a journal and its bearing. Indicate the distribution of pressure, and the position of the resultant pressure, between the surfaces. Describe how the principles involved are applied in the Michell type of thrust bearing. (*L.U.*)

18. A band-brake arrangement for a winch is shown in Fig. 62. Find the turning moment on the drum which can

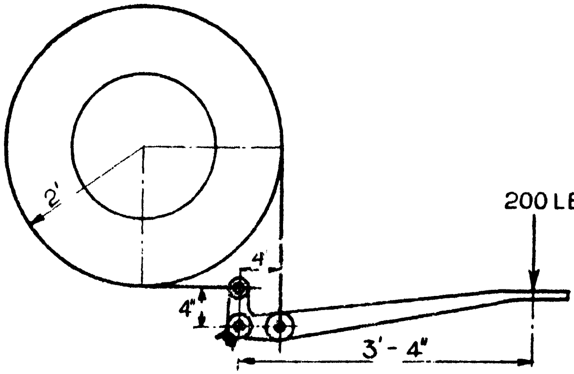

FIG. 62

be resisted by a force of 200 lb. applied vertically to the brake pedal. $\mu = 0 \cdot 10$. *Ans.* 925·6 lb. ft.

19. Give an account of the sources of loss which may occur in a simple belt drive between similar pulleys on parallel shafts and show that some give rise to a *loss of torque* and some to a *loss of speed*. Distinguish between *slip* and *creep*, and trace the history of a short section of the belt as it travels from a point somewhere on the slack side round the driven pulley to a point on the tight side. (*W.S.S.*)

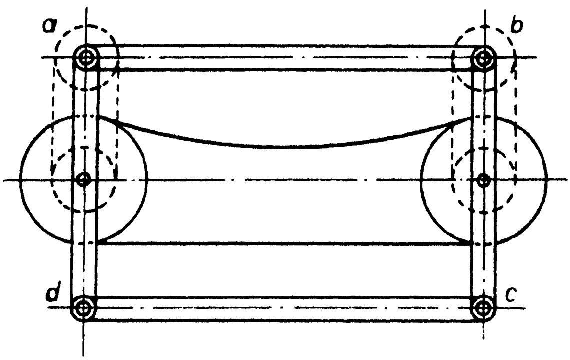

FIG. 63

20. A recent device for testing belts includes a mechanism represented in Fig. 63. It comprises an accurately constructed and free jointed parallelogram *abcd*, with one long

side (the upper one) fixed. Equal pulleys are mounted on ball bearings at the centres of the short sides; their axes are parallel and the belt to be examined is stretched between them. Power is transmitted to or away from these pulleys by the chains, and sprocket wheels shown dotted.

Given an accurately constructed and frictionless mechanism of this type, show how it could be used to determine (*a*) the loss in torque, and (*b*) the total loss in power in the belt drive. Indicate how the results obtained might be influenced by the friction in an actual testing machine of this type. (*W.S.S.*)

21. Write a short note on the advantages or disadvantages of (*a*) the bush roller chain and (*b*) the silent chain, as compared with belt drives for the transmission of power. Why is the silent type of chain superior in many respects to the bush roller type?

22. Two shafts distance c apart are connected by a chain drive, the chain wheels having N and n teeth respectively. Show that the chain length measured in pitches is

$$L = 2c + \tfrac{1}{2}(N + n) + (1/C)[(N - n)/2\pi]^2 .$$

23. Discuss whether a chain drive gives a uniform angular velocity ratio between two shafts and explain how a variation may arise. What are the chief causes of loss of efficiency in chain drives?

24. What shape should the teeth of a chain wheel be for (*a*) a roller chain, (*b*) a silent type chain? Given chains of these types, show how you would design suitable chain wheels.

8. TOOTHED GEARING—EPICYCLIC GEARS—CAMS

1. A pinion of 12 in. pitch-circle diameter is cut with involute teeth of 2 d.p. and 25° angle of obliquity. Draw the profile of one of the teeth. If the addendum height is 0·5 in. in this pinion and in the 48 in. wheel with which it meshes, find the angle that the pinion turns through while any one pair of teeth continue to maintain contact.

(I.M.E.)

SOLUTION. The setting out of the tooth profile is left as an exercise for the student. Let P (Fig. 64) be the pitch

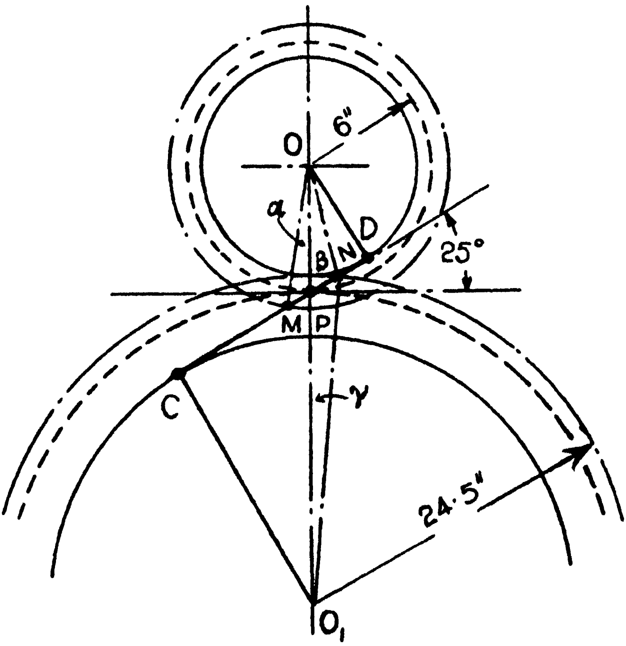

FIG. 64

point. Draw CD at 25° to the common normal at P. Then the path of contact is along CD.

With centre O and radius 6·5 in. draw an arc cutting CD in M. With centre O_1 and radius 24·5 in. draw a second arc

cutting CD in N. Then MN is the path of contact for any one pair of teeth and the pinion turns through an angle MON while one pair remain in contact.

Considering the triangles OMP, OPN, and O_1PN we have, $OP = 6$ in.; $O_1P = 24$ in.; $\angle MPO = 115° = \angle O_1PN$; $OM = 6{\cdot}5$ in.; $O_1N = 24{\cdot}5$ in.; $\angle OMP = \theta$; $\angle O_1NP = \phi$. Using the sine and cosine rules,

(i) $OM/\sin 115° = OP/\sin\theta$

$\therefore \quad \sin\theta = 6 \times 0{\cdot}9063/6{\cdot}5$, and $\theta = 56{\cdot}8°$

$\therefore \quad \alpha = [180° - (115° + 56{\cdot}8°)] = 8{\cdot}2°.$

(ii) $O_1N/\sin 115° = O_1P/\sin\phi$

$\therefore \quad \sin\phi = 24 \times 0{\cdot}9063/24{\cdot}5$, and $\phi = 62{\cdot}6°$

$\therefore \quad \gamma = [180° - (115° + 62{\cdot}6°)] = 2{\cdot}4°.$

(iii) $$ON^2 = OO_1{}^2 + O_1N^2 - 2(OO_1)(O_1N)\cos\gamma$$

$$ON^2 = (30)^2 + (24{\cdot}5)^2 - 2(30 \times 24{\cdot}5) \times 0{\cdot}9991$$

$\therefore \quad ON = 5{\cdot}619$ in.

(iv) $ON/\sin 2{\cdot}4° = O_1N/\sin\beta$

$\therefore \quad \sin\beta = 24{\cdot}5 \times 0{\cdot}0419/5{\cdot}619$, and $\beta = 10{\cdot}5°.$

The required angle $MON = \alpha + \beta = 8{\cdot}2° + 10{\cdot}5° = 18{\cdot}7°$

2. A gear-wheel with 40 involute teeth drives another with 20 teeth. The teeth are as long as possible and the angle of obliquity is 16°. Find the greatest sliding velocity between a pair of teeth when the linear velocity of the pitch lines is 10 ft./sec. (*I.C.E.*)

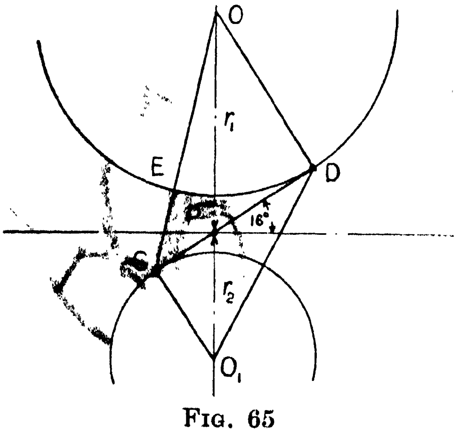

FIG. 65

SOLUTION. Let P (Fig. 65) be the pitch point and CD the path of contact.

To avoid interference and undercutting of the flanks of the teeth on the wheels the height of the

teeth on the larger base circle should not exceed the radial distance EC, since contact should not take place outside CD. Hence the maximum distance from the pitch point P, to a point of contact is $PD = r_1 \sin 16°$ where r_1 is the pitch circle radius of the larger wheel.

The velocity of sliding at any point of contact is given by

$v = (\omega_1 + \omega_2) \times$ distance from P to point of contact,

where ω_1 and ω_2 are the angular velocities of the respective wheels.

$$\therefore \qquad v_{max} = (\omega_1 + \omega_2)PD = (\omega_1 + \omega_2)r_1 \sin 16°.$$

If V is the linear velocity of the pitch line,

$$\omega_1 = V/r_1;\ \omega_2 = V/r_2$$

and hence

$$v_{max} = V(1/r_1 + 1/r_2) \cdot r_1 \sin 16°$$
$$= V(1 + r_1/r_2) \sin 16°.$$

Now $\qquad r_1/r_2 = \text{gear ratio} = 40/20 = 2$, and

$$V = 10 \text{ ft./sec.}$$

$$\therefore \qquad v_{max} = 10(1 + 2) \times 0{\cdot}2756$$
$$= 8{\cdot}268 \text{ ft./sec.}$$

3. Given the arcs of approach and recess for a pair of involute toothed wheels which mesh with one another show how the addendum circles may be obtained.

Geometrical Solution. If D (Fig. 66) is the point at which contact commences, the arc of approach is equal to either of the arcs PM or PL. M and L are the points at which the tooth profiles cut the pitch circles. Profile DM is the involute to the base circle A and if CM is a tangent to this circle we have

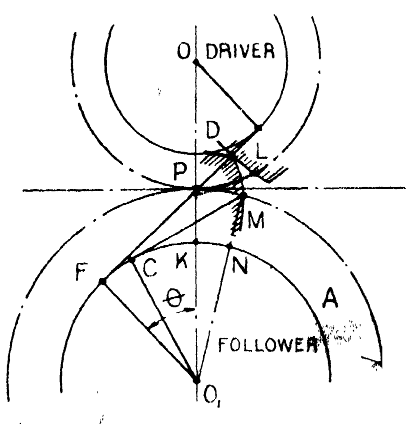

Fig. 66

$$\text{arc } CF + CM = FD$$

Now $FP = CM$ since both are tangents to the base circle and bounded by the pitch circle.

Hence $\qquad arc\ CF = PD.$

The triangles O_1FP and O_1CM are equal in all respects and so

$$arc\ CF = arc\ KN = PD.$$

It follows that in order to obtain the addendum circle of the follower when given the arc of approach, mark off PM on the pitch circle of the follower equal to the arc of approach. Join O_1M to meet the base circle of the follower in N. Along the path of contact set $PD = arc\ KN$. The circle through D with O_1 as centre is the required addendum circle.

A similar construction will give the addendum circle of the driver when the arc of recess is given.

4. Two gear-wheels of 4 in. and 6 in. pitch diameters have involute teeth of 8 diametral pitch and an angle of obliquity of 17°. The addenda are each $\frac{1}{8}$ in. Determine (*a*) the length of the path of contact; (*b*) the number of pairs of teeth in contact; (*c*) the angle turned through by the smaller wheel while any one pair of teeth are in contact. (*L.U.*)

Ans. (*a*) 0·7042 in.
(*b*) 3 pairs.
(*c*) 19·4°.

5. A pair of gear-wheels with involute teeth is to transmit 10 h.p. with a gear ratio of 5 to 1. The driving wheel is the smaller. It runs at 600 r.p.m. and is to have 15 teeth. The arc of approach is to be equal to the circumferential pitch. Assuming that $P = 850p^2$ where P is the tangential force at the pitch line in lb. and p is the circumferential pitch in inches, find—

(*a*) The distance between centres,
(*b*) The diametral pitch of the teeth, and
(*c*) The angle of obliquity if the line of action is as long as possible. (*I.C.E.*)

Ans. (*a*) 11·49 in.
(*b*) 3·916
(*c*) 22·7°

6. A flat frame like a door slides vertically in guides and is lifted by means of a pinion that meshes with a rack fitted centrally between the guides. The pinion has 20 involute teeth of 5 diametral pitch, 20° angle of obliquity, and is carried on a 2 in. shaft supported by a bearing, on each side. Assuming that the coefficient of friction on the guides and on the pinion bearings is 0·466 (= tan 25°), and neglecting friction on the pinion teeth, estimate the torque required in the pinion shaft to raise the frame weighing 2 cwt.

(*I.M.E.*)

Ans. 644·4 lb. in.

7. What is meant by the term *pitch point* as applied to two toothed wheels in mesh with one another? What is the relation between the pitch point and the direction of the tooth surfaces at the point of contact?

Suppose two wheels have accurate involute teeth, but that one of them is mounted a little eccentrically, what happens to the pitch point as the wheels revolve?

If the pitch diameters of the two wheels are 5 in. and 20 in. respectively, the larger wheel is eccentric by 0·01 in., and the angle of obliquity is 22°, calculate the maximum and minimum deviations from the nominal velocity ratio. Show how you would determine the deviation for any other point in a revolution and indicate by a sketched curve the periodic manner in which the deviation varies.

(*W.S.S.*)

Ans. Max. deviation 0·1001%.
Min. deviation − 0·0999%.

8. A gear-wheel with involute teeth is driving a larger wheel, the angle of obliquity being ϕ. For the driving wheel the radius of the base circle is R_b, of the pitch circle R_p, and of the addendum circle R_a. Show that the arc of recess measured along the pitch circle is $R_p (\tan \beta - \tan \phi)$, where $\beta = \cos^{-1} (R_b/R_a)$. Show, also, that when the thickness of a tooth at the tip is zero β has the value given by the equation,

$$\tan \beta - \beta = \tan \phi - \phi + \alpha$$

where $2\alpha R_p$ is the width of a tooth of the driver, measured at the circumference of the pitch circle.

(*Mech. Sc. Tripos Cam.*)

SOLUTION. The arc of recess depends on the addendum of the driver. If O (Fig. 67) is the pitch point and AB the path of contact, then P, the point where the addendum circle of

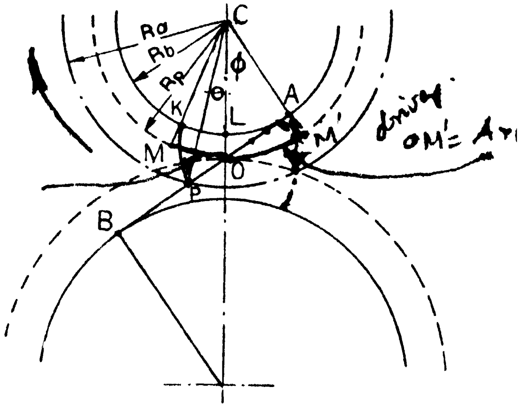

FIG. 67

the driver cuts the line AB, is the point at which contact ceases. OM is then the arc of recess and it has been shown, in Problem 3, that the arc KL on the base circle is equal to the length OP.

Let a, the arc of recess, $= OM$.

Since $OP = KL$ we have

$$OP/\text{arc } OM = CL/CO = CA/CO = \cos\phi = R_b/R_p$$

or $$\text{arc } OM = a = OP/\cos\phi. \qquad \text{(i)}$$

Considering the triangles CPA and COA

$$(AP)^2 = (CP)^2 - (CA)^2 = R_a^2 - R_b^2$$

also $$AO = CO\sin\phi = R_p\sin\phi$$

$$\therefore \quad OP = AP - AO = \sqrt{(R_a^2 - R_b^2)} - R_p\sin\phi$$

$$\therefore \quad a = [\sqrt{(R_a^2 - R_b^2)} - R_p\sin\phi]/\cos\phi$$

$$= R_p\left[\frac{\sqrt{(R_a^2 - R_b^2)}}{R_p\cos\phi} - \tan\phi\right].$$

Now from (i) $R_p \cos \phi = R_b$ and so

$$a = R_p\{\tan \beta - \tan \phi\} \text{ where}$$
$$\beta = (\phi + \theta) = \cos^{-1} R_b/R_a$$
$$= \tan^{-1}[(1/R_b)\sqrt{(R_a{}^2 - R_b{}^2)}].$$

The second part of the problem is left to the student.

9. Write a brief note on the relative advantages and disadvantages of involute and cycloidal teeth for gear-wheels. Construct the cycloidal teeth for a pair of wheels with 5 in. and 12 in. pitch circle diameters

(*a*) when they gear internally, and
(*b*) when they gear externally.

10. In the case of a pair of spur-wheels having involute teeth show that, during the time that two pairs of teeth are in contact, the efficiency of transmission is constant and equal to $(2Rr \cos \theta - \mu ar)/(2Rr \cos \theta + \mu ar)$ assuming that the normal pressure between each pair is the same; where

r is the pitch circle radius of the driver,
R is the pitch circle radius of the follower,
μ is the coefficient of friction,
a is the distance between the points of contact,
θ is the angle of obliquity.

(*Mech. Sc. Tripos Cam.*)

11. A worm gear to transmit 40 h.p. with a speed reduction from 1 000 to 100 r.p.m. would normally have a worm with a multiple thread. Give reasons for this, and suggest suitable values for the number of threads on the worm and the number of teeth in the wheel, in the above case.

(*W.S.S.*)

12. Explain carefully why in such a gear as that in the previous problem, the cross-section of a wheel-tooth made by the central plane of the wheel has a different profile from the section made by any other parallel plane. Describe some method of cutting the wheel. (*W.S.S.*)

13. A reduction gear for a total ratio of 60 : 1 is arranged with a first step of 15 :1 by worm-gear and a second step of 4 : 1 by spur gearing. A double worm arrangement is used with similar right- and left-hand worms, triple threaded, 4 in. diameter, $\frac{3}{4}$ in. pitch cut on the same spindle.

These are in gear with two worm-wheels of 45 teeth arranged, respectively, above and below the worms. Each worm-wheel shaft carries a pinion and these both mesh with the main gear-wheel. Assuming a square worm thread with a coefficient of friction 0·05, and taking an efficiency of 93% for the spur gears, calculate the torque acting on the main gear-wheel shaft when the worm spindle receives 15 h.p. at 900 r.p.m. (*L.U.*)

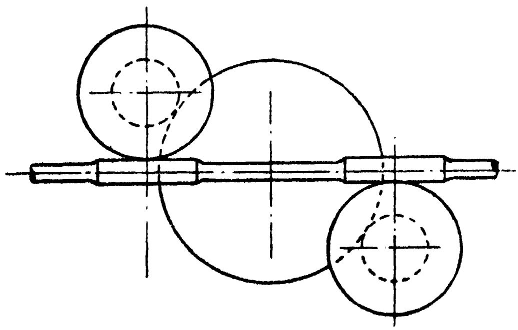

FIG. 68

SOLUTION (Fig. 68). The efficiency of the worm-gears is given by

$$\eta = \tan\theta/\tan(\theta + \phi)$$

where θ = thread angle of worm

$\tan\phi = \mu$ = coefficient of friction.

$$\text{Now}\quad \tan\theta = \frac{\text{lead}}{\text{pitch circumference}}$$

$$= \frac{\text{pitch} \times \text{number of threads}}{\text{pitch circumference}}$$

$$\therefore\quad \tan\theta = (0{\cdot}75 \times 3)/4\pi = 0{\cdot}1791 \text{ and } \theta = 10{\cdot}2^\circ$$

$$\tan\phi = 0{\cdot}05 \quad \therefore\ \phi = 2{\cdot}9^\circ$$

$$\therefore\quad (\theta + \phi) = 13{\cdot}1^\circ \text{ and } \tan(\theta + \phi) = 0{\cdot}2327.$$

Hence the efficiency of the worm-gear is

$$\eta = 0{\cdot}1791/0{\cdot}2327 = 76{\cdot}96\%.$$

Assuming that each worm receives 7·5 h.p., then each delivers 7·5 × (76·96/100) = 5·773 h.p. to the spur gears, and these in turn each deliver 5·773 × (93/100) = 5·37 h.p. to the main gear-wheel. Thus the total power received by this wheel is

$$5{\cdot}37 \times 2 = 10{\cdot}74 \text{ h.p.}$$

The speed of the main gear-wheel is

$$900/60 = 15 \text{ r.p.m.}$$

and we have $\quad$ h.p. $= 2\pi NT/33\,000$

$\therefore \quad T = 33\,000 \times (\text{h.p.})/2\pi N$

In this case T = torque $= 33\,000 \times 10{\cdot}74/(2\pi \times 15)$

$$= 3\,761{\cdot}6 \text{ lb. ft.}$$

14. A worm reduction gear transmits 30 h.p., with speeds of worm and wheel at 1 600 and 40 r.p.m., respectively. The worm is $3\frac{1}{2}$ in. diameter, 1 in. pitch, single threaded. The thrust is taken by a flat collar bearing, mean diameter 5 in. The coefficient of friction for worm and thrust may be taken as 0·05. Determine the efficiency of the gear, and show whether the gear is self-locking if the drive is attempted from the wheel side. *Ans.* 42·4%.

Note. Problems on epicyclic gears generally call for velocity ratios between driving and driven elements. For solving such problems the tabular method is recommended and has been adopted in the worked examples which follow.

15. The diagram (Fig. 69) shows a simple epicyclic gear. The wheel *A* is stationary. What is the speed of *C* when *B* rotates about its own axis at 100 r.p.m.? The numbers of teeth on *A* and *B* are 120 and 45 respectively.

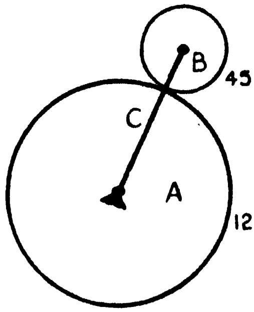

FIG. 69

SOLUTION. Make out a table as below—

REMARKS	*A*	*B*	*C*	REMARKS
C fixed and *A* free to rotate	$-\frac{9}{24}$	1	0	Giving *B* 1 turn
Add $\frac{9}{24}$ right across	0	$\frac{33}{24}$	$\frac{9}{24}$	This fixes *A*
Simplifying	0	33	9	
$\times \frac{100}{33}$ right across	0	100	$\frac{300}{11}$	Thus giving *B* 100 turns

Hence we see that when B rotates at 100 r.p.m. about its axis, then C rotates at $\frac{300}{11}$ r.p.m. in the same direction.

16. In the gear shown in the diagram (Fig. 70) the annular wheel B is fixed. What is the velocity ratio of shaft C to shaft A?

SOLUTION.

REMARKS	A	B	C	REMARKS
C fixed and B free to rotate	1	$-\frac{7}{11}$	0	Giving A 1 turn
Add $\frac{7}{11}$ right across	$\frac{18}{11}$	0	$\frac{7}{11}$	This fixes B
Simplifying	18	0	7	

When A makes 18 revolutions C makes 7 revolutions in the same direction, hence the required velocity ratio is

$$\text{Speed of } A/\text{Speed of } C = 18/7$$

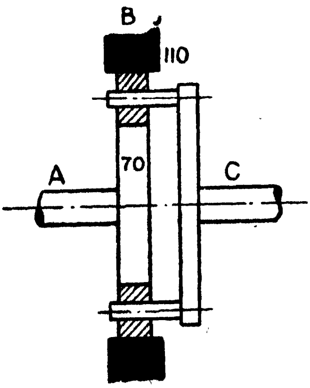

FIG. 70

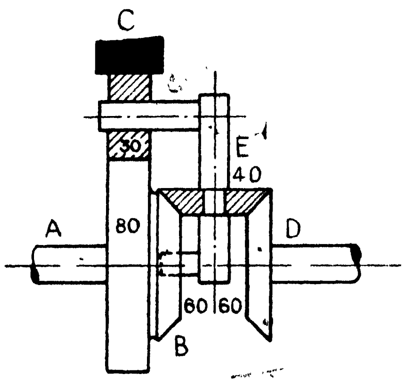

FIG. 71

17. In the gear shown (Fig. 71) the wheel C is fixed and the wheel A rotates at 250 r.p.m. The bevel wheel B is rigidly fastened to A. Find the speed of the shaft D.

SOLUTION. (i) Consider the train A, C and the arm E.

REMARKS	A	C	E	REMARKS
E fixed and C free to rotate	1	$-\frac{4}{7}$	0	Giving A 1 turn
Add $\frac{4}{7}$ right across	$\frac{11}{7}$	0	$\frac{4}{7}$	This fixes C
Simplifying	11	0	4	

When A makes 11 turns, E makes 4 turns in the same direction.

(ii) Considering the train B, E and D.

REMARKS	B	E	D	REMARKS
The problem is	11	4	?	
E fixed as in (i)	1	0	-1	Giving B 1 turn (a)
Subtract 1 right across	0	-1	-2	This fixes B (b)
Multiply (a) by 11	11	0	-11	This makes B, 11 and E, 0
Multiply (b) by -4	0	4	8	This makes B, 0 and E, 4
Add last two lines	11	4	-3	This makes B, 11 and E, 4
Multiply by $\frac{250}{11}$ right across	250	$\frac{1000}{11}$	$-\frac{750}{11}$	

Therefore when A makes 250 r.p.m., D makes 750/11 r.p.m. in the opposite direction.

18. The diagram (Fig. 72) shows an epicyclic arrangement for a winding gear. Shafts A and B are driven by separate motors. If B has a constant speed of 750 r.p.m., and N

indicates the r.p.m. of A, the positive sign denoting the same sense of rotation as B, find an expression giving the speed of the drum attached to C in terms of N, and hence find the speed of the drum when N = 1 500.

Ans. 1 559·35 r.p.m.

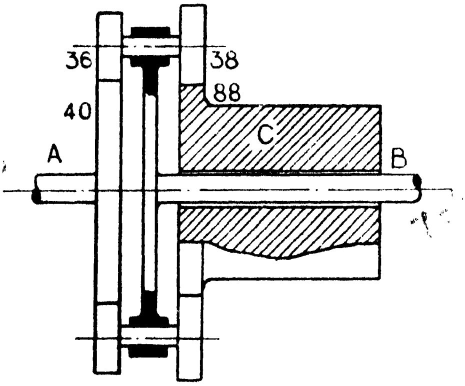

FIG. 72

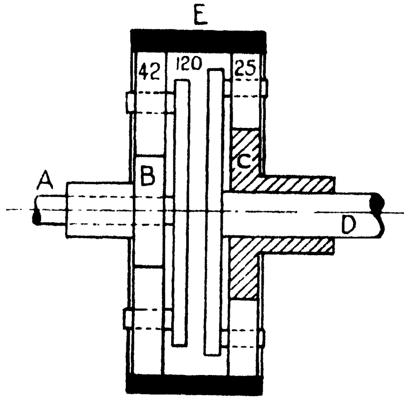

FIG. 73

19. Find the velocity ratio of the epicyclic gear in Fig. 73 when B is the driving shaft. A and C are stationary, and D is the driven shaft.

Ans. Velocity ratio = $\dot{-}$ 5·28.

20. Power is transmitted from shaft A to shaft B of the gearbox shown in Fig. 74. It is found that a torque of

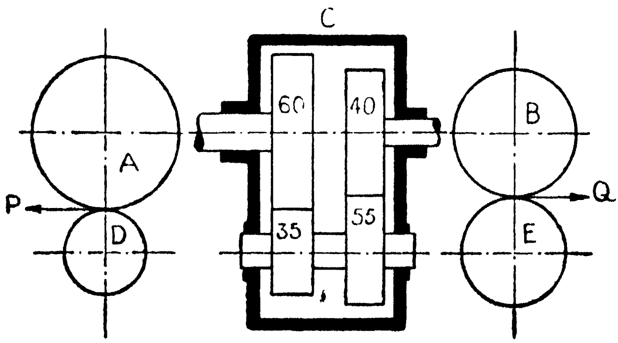

FIG. 74

500 lb. ft. must be applied to the casing C about the axis AB, to hold it stationary. If A rotates at 1 000 r.p.m., find

the power transmitted, neglecting friction and assuming no backlash between the toothed wheels.

SOLUTION. When the casing is fixed the speed of B is given by

$$1\,000 \times \frac{60}{35} \times \frac{55}{40} = \frac{33\,000}{14} = \frac{16\,500}{7} \text{ r.p.m. in same direction.}$$

Let P = pressure between teeth of A and D

Q = pressure between teeth of E and B.

Then torque tending to turn casing is evidently

$$T_C = (P \times r_A) + (Q \times r_B) = T_A + T_B$$

Now torque on $A = T_A = \dfrac{(\text{h.p.}) \times 33\,000}{2\pi \times 1\,000}$

and torque on $B = T_B = \dfrac{(\text{h.p.}) \times 33\,000 \times 7}{2\pi \times 16\,500}$

Hence

$$\begin{aligned} T_C &= T_A + T_B \\ &= \frac{(\text{h.p.}) \times 33\,000}{2\pi}\left(\frac{1}{1\,000} + \frac{7}{16\,500}\right) \\ &= 500 \text{ lb. ft.} \end{aligned}$$

$$\therefore \text{ H.p. transmitted} = \frac{2\pi \times 500 \times 1\,000 \times 16\,500}{33\,000 \times 23\,500} = 66{\cdot}8 \text{ h.p.}$$

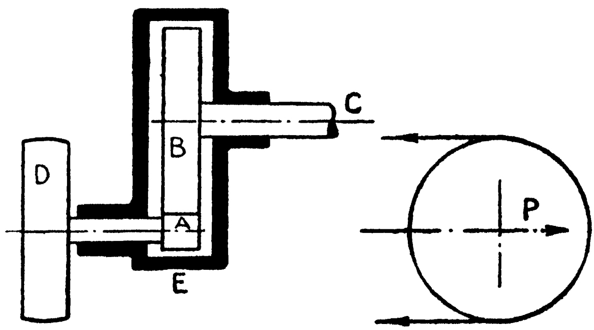

FIG. 75

21. The diagram (Fig. 75) gives the arrangement of a recent automatic belt tensioner. The shaft C is part of the

machine being driven, and is driven by a belt through the pulley D and the pinions A and B. The casing E can swing about the shaft C. Show that the belt will automatically tighten as the power delivered to D increases, and find a general expression for the force P tending to tighten the belt. Take a and b as the numbers of teeth in A and B respectively and T as the torque on D.

Ans. $P = 2T/(a + b)$.

22. The differential gears shown in Fig. 76 are part of an epicyclic gearbox. The annular wheel C is held stationary.

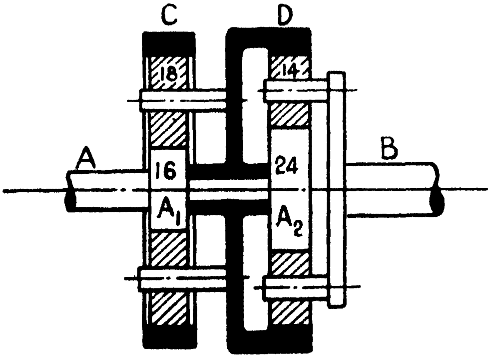

FIG. 76

If A is the driving shaft and B the driven, find the velocity ratio of the arrangement.

Ans. 323/154.

23. If the power transmitted by the gearbox in the previous problem is 10 h.p. and the speed of B is 1 000 r.p.m., find the torque on the shaft A and the proportions of this torque transmitted by the pinions A_1 and A_2 respectively. Assume no backlash or frictional losses.

SOLUTION.

$$\text{H.p.} = 2\pi \times N_A \times T_A/33\,000$$

$$\therefore \qquad T_A = 33\,000 \times 10/2\pi N_A \text{ lb. ft.}$$

The velocity ratio $= N_A/N_B = 323/154$

$$\therefore \qquad N_A = (323/154) \times 1\,000 = 323\,000/154 \text{ r.p.m.}$$

Hence $\quad T_A = 33\,000 \times 10 \times 154/2\pi \times 323\,000$

$$= 25{\cdot}05 \text{ lb. ft.}$$

To find the manner in which this torque is divided between the pinions A_1 and A_2, consider the tangential forces at the pitch circles of the trains (Fig. 77).

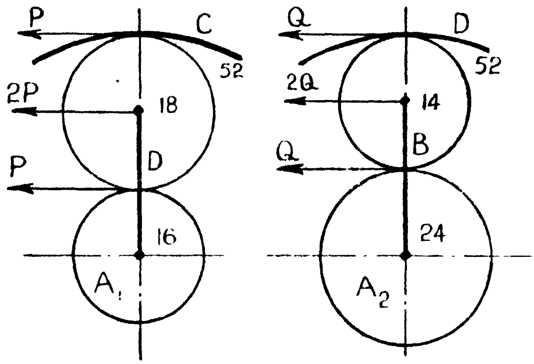

FIG. 77

Left-hand train A_1, C, and D.

$T_1 = 8P$ taking pitch diameter as proportional to number of teeth. . . . . (1)

Right-hand train A_2, D, and B.

$T_2 = 12Q$. . . . . (2)

Equating the moments on the casing D due to forces between teeth,

$$2P \times 17 = Q \times 26$$

$$\therefore \quad Q = (17/13)P \quad . \quad . \quad . \quad (3)$$

Substituting (3) in (2)

$$T_2 = (12 \times 17/13)P \quad . \quad . \quad . \quad . \quad (4)$$

Now $\quad T_A = (T_1 + T_2)$ and using (1) and (4).

$$\therefore \quad T_1 = \frac{T_1}{(T_1 + T_2)} \cdot T_A = \frac{8P}{[8 + (12 \times 17)/13]P} \cdot T_A$$

$$= \frac{26}{77} \cdot T_A.$$

$$T_2 = \frac{T_2}{(T_1 + T_2)} \cdot T_A = \frac{(12 \times 17)/13 \, . \, P}{[8 + (12 + 17)/13]P} \cdot T_A$$

$$= \frac{51}{77} \cdot T_A.$$

Therefore the torque T_A on A is divided between A_1 and A_2 in the ratio,

$$T_1/T_2 = 26/51.$$

24. The diagram (Fig. 78) shows an arrangement of two parallel shafts driven by equal wheels from the shaft A. At the ends remote from this shaft the parallel shafts are

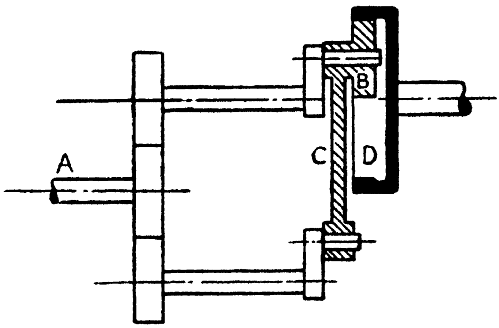

Fig. 78

coupled together by equal overhung cranks and by a connecting link C. Solid with this link is a toothed wheel B, its axis coinciding with that of the upper crank-pin. This wheel gears internally with a wheel D in line with the upper shaft. If the speed of A is n r.p.m. and B and D have 14 and 32 teeth respectively, show that the speed of D is $n/16$ r.p.m. in the direction opposite to that of the shaft A.

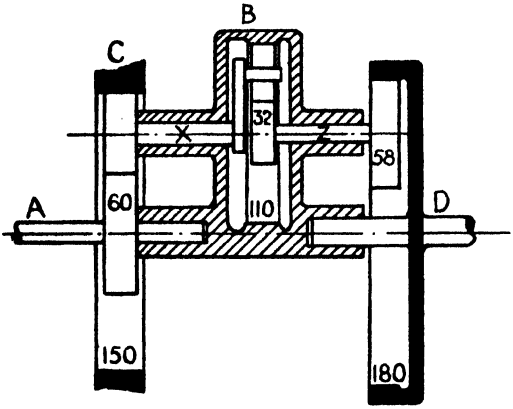

Fig. 79

25. A double epicyclic gear is shown in Fig. 79. The annular wheel C is held stationary and the casing B contains an epicyclic gear which gives motion to the shaft Z. Find the velocity ratio of A and D.

(*Hint.* Find speed of shaft X about its axis and use this speed to solve the differential within the casing B, thus getting speed of shaft Z about its axis.)

Ans. – 33/1.

26. If the casing B in the previous problem were held at rest and C were free to rotate, what would be the velocity ratio of A to D, and what torque would be necessary to keep B at rest if A receives 10 h.p. at 750 r.p.m.? Neglect friction.

Ans. – 1·9 and 33·15 lb. ft.

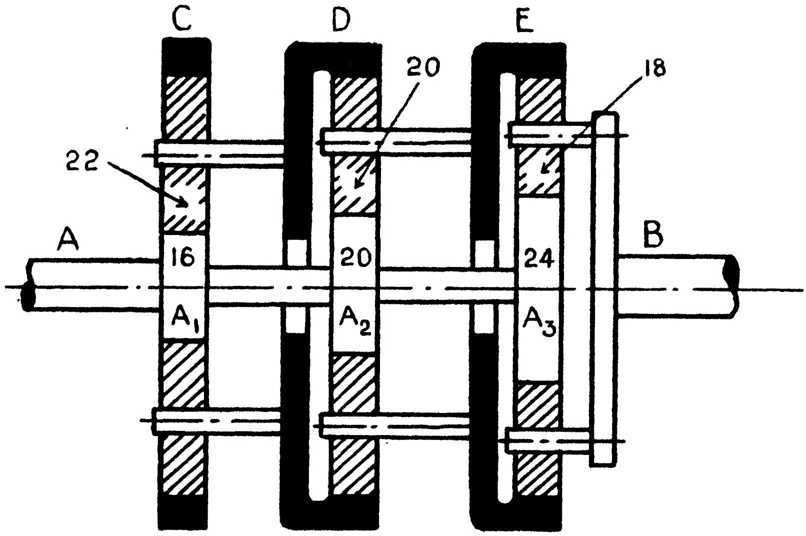

FIG. 80

27. Find the velocity ratios of A to B for the epicyclic gear shown in Fig. 80, when C, D and E are fixed in turn.

Ans. 1·73; 3·5; – 14.

28. If T_A is the torque applied to A_1 and C is stationary in Problem 27, show that $(60/281) \cdot T_A$, $(85/281) \cdot T_A$ and $(136/281) \cdot T_A$ are the proportions of this torque transmitted by the wheels A_1, A_2 and A_3, respectively. Neglect backlash and friction.

29. The diagram (Fig. 81) shows an arrangement of the Wilson easy-change gearbox to give four forward speeds

and one reverse speed of the shaft F. A is the driving shaft. Find the velocity ratios of A to F when B, C, D and E are

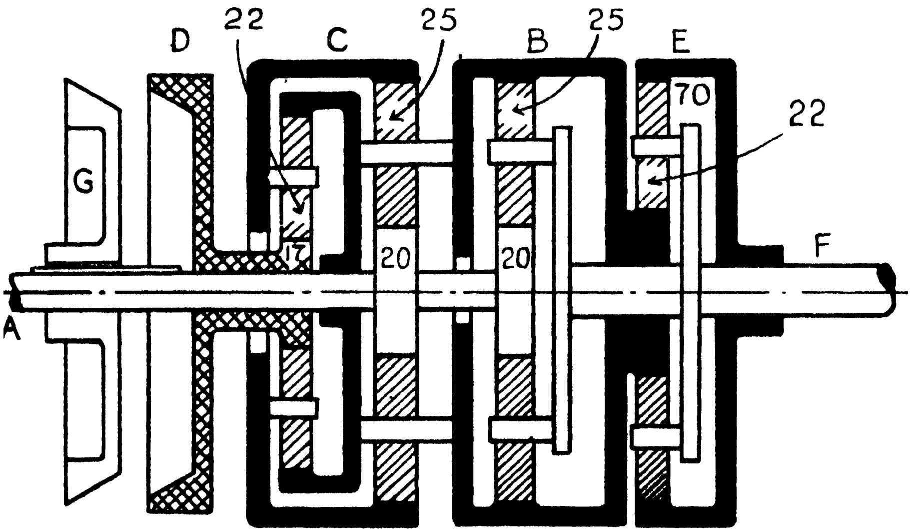

FIG. 81

fixed in turn, and when the clutch G is fixed to D. G moves along A on a key.

Ans. 4·5; 2·53; 1·51; − 8·42; 1.

30. The diagram (Fig. 82) gives the profile of a cam that rotates about the axis O and lifts the block P. Draw a graph to show the lift of the block plotted on a base representing the angle turned by the cam. Also, calculate the acceleration of the block at the top of its lift when the cam rotates at 120 r.p.m., and mark the profile of the cam to show where you would expect to find wear. (*I.M.E.*)

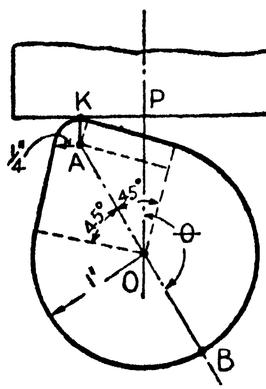

FIG. 82

SOLUTION.

$$OA = \sqrt{[(3/4)^2 + (3/4)^2]}$$
$$= \sqrt{2} \times 3/4 = 1{\cdot}06 \text{ in.}$$

Measuring the angle turned through by the cam by the position of the radius OB relative to the line of lift OP, we

see that the block begins to lift when $\theta = 135°$, continues to lift until $\theta = 180°$, and falls again until $\theta = 225°$. The lift during the period $\theta = 135°$ to $\theta = 225°$, is given by,

$$y = OP - OB$$
$$= OA \cos (180 - \theta)° + AK - OB$$
$$= -1{\cdot}06 \cos \theta + 0{\cdot}25 - 1$$
$$\therefore \quad y = -(1{\cdot}06 \cos \theta + 0{\cdot}75) \text{ in.}$$

If this is plotted from $\theta = 135°$ to $\theta = 225°$ the required lift curve is obtained.

During this period the acceleration of the block is given by d^2y/dt^2, obtained by differentiating y twice with respect to time.

Thus $\quad dy/dt = -(1{\cdot}06 \times -\sin \theta)(d\theta/dt) = 1{\cdot}06\omega \sin \theta$,

since $\quad d\theta/dt = \omega$.

And so $\quad d^2y/dt^2 = 1{\cdot}06\omega \cos \theta \,.\, (d\theta/dt) = 1{\cdot}06\omega^2 . \cos \theta$.

Now $\quad \omega = (2\pi \times 120)/60 = 4\pi$ radn./sec.

At the top of the lift $\theta = 180° \therefore \cos \theta = -1$ and so the acceleration at this point is

$$d^2y/dt^2 = 1{\cdot}06 \times (4\pi)^2 \times -1 = -167{\cdot}3 \text{ in./sec.}^2$$

The negative sign indicates a retardation, which is in keeping with the nature of the motion.

31. The figure shows a circular cam C that rotates with the shaft S and lifts a roller R carried by a slider L that moves in guides G. It is required to modify the cam C so that the motion of the slider L shall be as nearly as possible simple-harmonic when the shaft turns with constant angular velocity. Mark off the profile to show how the existing cam should be cut down, to give the same stroke with the same roller. (*I.M.E.*)

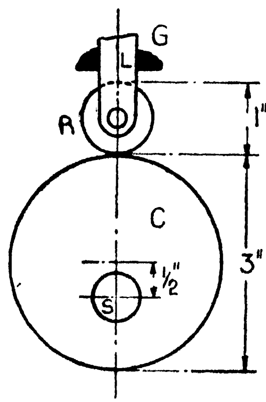

FIG. 83

32. The least radius of a cam is ½ in. and its profile is an involute drawn to a base circle of 1 in. diameter. The cam

actuates a follower which is guided to move in a line passing through the centre of rotation of the cam and which is provided with a roller $1\frac{1}{2}$ in. diameter. What angular movement of the camshaft is required to move the follower $\frac{3}{4}$ in. from the beginning of its out-stroke? (*I.C.E.*)

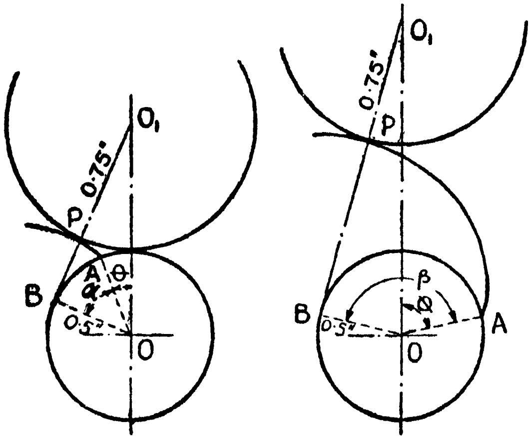

FIG. 84

SOLUTION. In Fig. 84 are shown the positions of cam and follower at the beginning of the out-stroke. We have

$$OO_1 = 1{\cdot}25 \text{ in.}; \quad OB = 0{\cdot}5 \text{ in.} \quad O_1P = 0{\cdot}75 \text{ in.}$$

$\therefore \quad O_1B = 1{\cdot}146$ in.

and so $\quad BP = (1{\cdot}146 - 0{\cdot}75) = 0{\cdot}396$ in.

Now since the cam profile is an involute,

$$\text{Arc } BA = BP = 0{\cdot}396 \text{ in.}$$

$\therefore \quad 0{\cdot}396 = 0{\cdot}5 \times \alpha$, whence $\alpha = 0{\cdot}396/0{\cdot}5$
$= 0{\cdot}792$ radn.

Also, $\cos(\alpha + \theta) = 0{\cdot}5/1{\cdot}25 = 0{\cdot}4$

$\therefore \quad (\alpha + \theta) = 1{\cdot}1589$ radn.

So that $\quad \theta = (1{\cdot}1589 - 0{\cdot}792) = 0{\cdot}3669$ radn.

The positions when the lift is $\frac{3}{4}$ in. are shown in the second half of the diagram, and we now have

$$OO_1 = (0{\cdot}75 + 0{\cdot}75 + 0{\cdot}5) = 2 \text{ in.}$$
$$OB = 0{\cdot}5 \text{ in.}$$

$$\therefore \qquad O_1B = 1{\cdot}936 \text{ in.}$$
$$\text{and} \qquad BP = (1{\cdot}936 - 0{\cdot}75) = 1{\cdot}186 \text{ in.}$$

As before,

$$\text{Arc } BA = BP = 1{\cdot}186 \text{ in.}$$
$$\therefore \qquad 1{\cdot}186 = 0{\cdot}5 \times \beta, \text{ and } \beta = 1{\cdot}186/0{\cdot}5$$
$$= 2{\cdot}372 \text{ radn.}$$
$$\cos(\beta - \phi) = 0{\cdot}5/2 = 0{\cdot}25$$
$$\therefore \qquad (\beta - \phi) = 1{\cdot}3177 \text{ radn.}$$
$$\text{Hence} \qquad \phi = (2{\cdot}372 - 1{\cdot}3177) = 1{\cdot}0543 \text{ radn.}$$

The total angular movement of the camshaft is

$$(\theta + \phi) = (0{\cdot}3669 + 1{\cdot}0543)$$
$$= 1{\cdot}4212 \text{ radn.}$$

33. A circular cam of radius r and eccentricity e operates an oscillating lever, the axes of camshaft and lever being parallel to one another and separated by a distance l. The contact surface of the lever is part of a plane parallel to the two axes, passing between them at a distance r_l from the axis of the lever. l is considerably greater than $r + r_l$. Find the extreme positions of the line of contact on the plane surface of the lever. (*W.S.S.*)

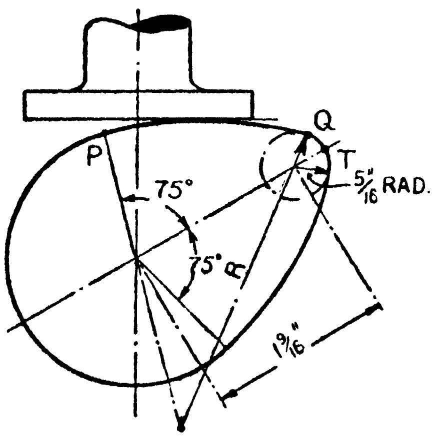

FIG. 85

34. Particulars of a cam with a flat-footed follower are given in Fig. 85. Calculate the value of the radius R, and

plot on a base of $3\frac{3}{4}$ in. = 150° a curve of displacement of the follower. If the speed of the cam is 500 r.p.m., calculate the accelerations of the follower at the points P, Q, and T. And if the combined mass of the follower and valve with which it is in contact is W lb., show how to calculate the stiffness of the spring which is employed to close the valve.

(*L.U.A.*)

Ans. R = 2·864 in.

35. Design a cam for a horizontal shaft, which will give an oscillatory motion of magnitude 2 in. to a vertical spindle provided with a roller of diameter 1 in. The spindle is to rise from its lowest to its highest point with simple harmonic motion in one-third of a revolution of the cam; there is to be a period of rest corresponding to the second third of a revolution, and a descent to the original position with constant acceleration during the remaining third of a revolution.

The spindle is 2 in. distant from the centre line of the camshaft, and when in its lowest position the centre of the roller is two inches above the plane containing the axis of the camshaft.

36. Design a cam to lift a spindle vertically at uniform speed through $2\frac{1}{2}$ in. and to return it at a uniform speed of half the lifting speed. The spindle is fitted with a roller $\frac{1}{2}$ in. diameter and the line of stroke is $\frac{1}{4}$ in. from the axis of the cam. Least radius of cam 2 in.

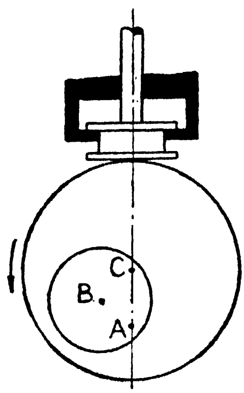

FIG. 86

37. A fuel injection pump of variable stroke is operated by a compound cam controlled by a governor. Reduced to its essentials, the mechanism is shown in Fig. 86. A, B, and C are the respective centres of the shaft which operates the pump, a disc fixed to the shaft, and an eccentric sheave mounted on the disc. The sheave can be turned round the disc by the governor mechanism and in this way the throw of the cam can be changed. The stroke of the pump is small compared with the total throw of the cam, and the return

stroke, which is effected by means of a spring, is limited by the stops shown in the figure.

AB and *BC* are each equal to 1 in. and the stroke of the pump is $\frac{1}{4}$ in. when the angle *ABC* is 120°. Through what angle must the sheave be turned in order to reduce the stroke to $\frac{1}{8}$ in.? By what fraction of a revolution of the shaft will the period of injection be retarded? (*W.S.S.*)

Ans. 13°.
0·0233 of a revolution.

38. The diagram (Fig. 87) shows the mechanism known as Kermode's steering gear in its mid-position. *A* is a spur wheel (driven by the steering engine) concentric with the rudder head and turning freely on it. In *A* is a curved slot *B* in which the pin *M* slides. This pin *M* is constrained to move in a straight line by guides *L* placed transversely across the ship. The rudder head is actuated from *M* by a connecting link *C* as shown.

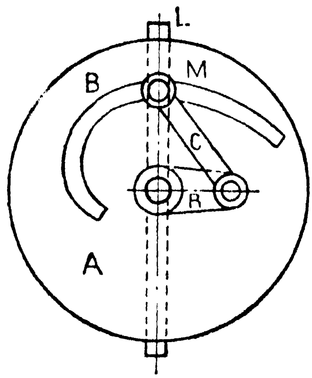

FIG. 87

If the length of the link *C* in a particular case is 4 ft. and the radius of the tiller is 2 ft., find the shape of the slot *B* so that the rudder may turn through equal angles as *A* rotates through equal angles.

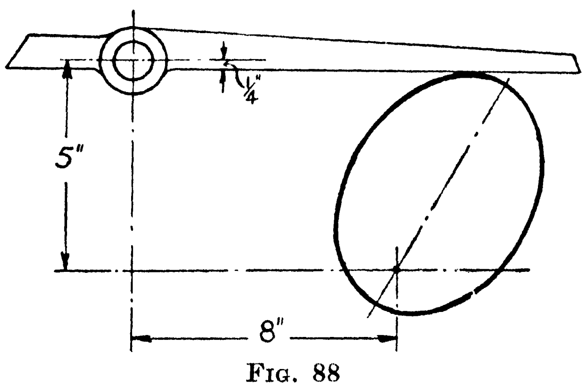

FIG. 88

39. The diagram (Fig. 88) shows an arrangement of an elliptic cam rotating about a focus, and operating a rocker

arm. If the cam rotates at a uniform speed of 200 r.p.m. plot curves showing the angular velocity of the rocker arm on a time base. The axes of the ellipse are 6 in. and 4 in. respectively.

40. A steam engine slide valve is to have a mean cut-off at 60% stroke with a steam lead of $\frac{1}{16}$ in., and a maximum opening to steam of $\frac{3}{4}$ in. Find the necessary travel of the valve.

This valve, instead of being driven by an eccentric is to be operated with the same motion by a cam running at engine speed. It is to be actuated by a roller, 1 in. diameter, mounted on a rod in line with the centre of the camshaft, the return drive being spring operated.

Draw the half of the cam contour on one side of the line of symmetry, given that the minimum distance between the centres of the cam and roller is $1\frac{1}{4}$ in. *(L.U.)*

The resultant reactions are obviously greatest when the c.g. is vertically below the axis of rotation, and least when it is vertically above. Maximum reactions are—

L.h. bearing (27 790 + 980) = 28 770 lb.

R.h. bearing (35 730 + 1 260) = 36 990 lb.

Minimum reactions are—

L.h. bearing (27 790 − 980) = 26 810 lb.

R.h. bearing (35 730 − 1 260) = 34 470 lb.

The bending moment diagrams are left as an exercise.

20. The diagram (Fig. 101) shows three rotating weights

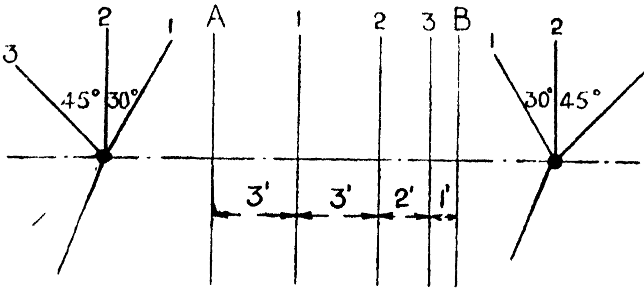

FIG. 101

of 400, 600 and 500 lb. at radii of 1, 1·5, and 2 ft. respectively. Find the balance weights required in the planes A and B for complete dynamic balance.

SOLUTION. Since the rotating weights are in different planes they give rise to an out-of-balance couple as well as an out-of-balance force. To balance such a system we require two rotating balance weights, one in each of two different planes, such as A and B above. The conditions to be satisfied are, (i) the force polygon must close, i.e. $\Sigma Wr = 0$, (ii) the couple polygon must close, i.e. $\Sigma Wrx = 0$, the moments being taken about any plane.

Thus to find the force required in the plane B, we choose A as a reference plane and draw the couple polygon (taking moments about A).

It is perhaps advisable to tabulate the values of Wr, and Wrx for the various weights.

			A	*B*
WEIGHT	RADIUS	Wr	Wrx	Wry
400 lb.	1·0 ft.	400	1 200	2 400
600 lb.	1·5 ft.	900	5 400	2 700
500 lb.	2·0 ft.	1 000	8 000	1 000

The couple polygon with A as reference plane is given in Fig. 102 (a). The closing line scales 13 140 so that the value of Wr for the plane B is 13 140/9 = 1 460.

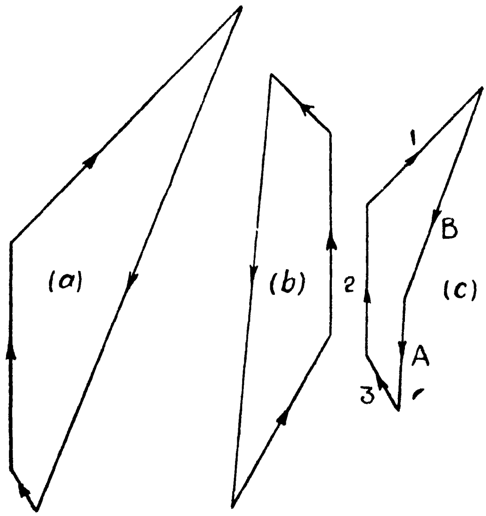

FIG. 102

Next, taking B as a reference plane, we draw the couple polygon (taking moments about B) as shown in Fig. 102 (b). The closing line scales 5 310 so that Wr for the balance weight in plane A is 5 310/9 = 590. The angles are scaled off the diagrams, and the accuracy of the work may be checked by drawing the force polygon as in Fig. 102 (c).

Hence, a balance weight in plane B of 1 460 lb. at 1 ft.

radius, making 127° with the direction of No. 1, and a balance weight in plane A of 590 lb. at 1 ft. radius, making 129° with the direction of No. 3, would be suitable.

21. Three rotating weights are equally spaced 30 in. apart along a shaft and all at radii of 10 in. The angle between 1 and 2 is 120° and between 2 and 3, 120°. The three weights are 150 lb. each. Find the magnitude of two balance weights at 15 in. radius, and their position in planes $7\frac{1}{2}$ in. from 1 and 3 respectively (outside) to produce complete dynamic balance.

Ans. 69·2 lb. each at 30° to plane of 1.

22. A uniform rectangular plate, shown in Fig. 103, rotates about a diagonal. Find the position and magnitude

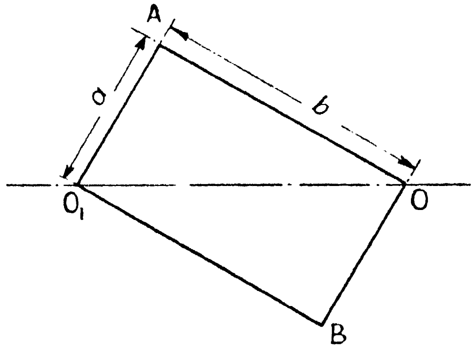

FIG. 103

of the least weights which when attached to the sloping edges OA and O_1B will produce complete balance, if W is the total weight of the plate.

Ans. $(2/9)Wb^2(b^2 - a^2)/(b^2 + a^2)$ at $(b^2 + a^2)/4b$ from O and O_1.

23. Distinguish between the primary and secondary force due to a reciprocating mass and state the conditions to be fulfilled for the mutual primary balance of a set of reciprocating masses. In a four-crank engine the distances between the cylinders are equal. Three reciprocating masses reckoned in succession from the right are $1\frac{1}{2}$, 2 and $2\frac{1}{2}$ tons. Find the fourth reciprocating mass and the crank angles relative to the right-hand crank for mutual primary balance.

SOLUTION. The inertia force due to a reciprocating weight W for any crank angle θ is given approximately by $(W/g)\omega^2 r[\cos\theta + (\cos 2\theta)/n]$ where $n = l/r$ the ratio of connecting-rod length to crank throw. This is the force which has to be balanced, and the first term—$(W/g)\omega^2 r.\cos\theta$—represents the projection, on to the line of stroke, of the

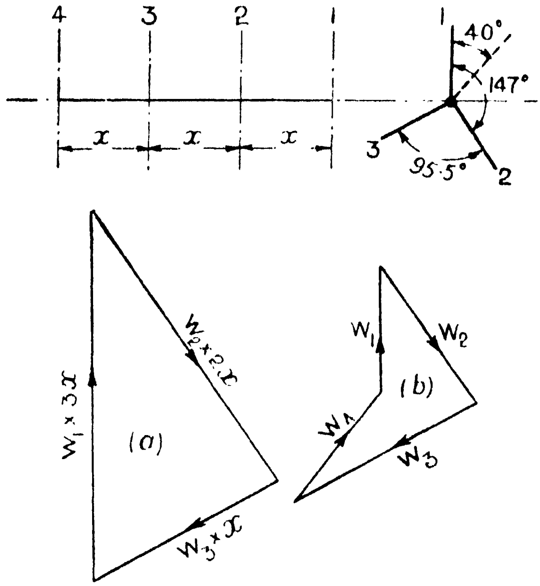

FIG. 104

inertia force due to a weight W, concentrated at the crank-pin, and rotating at an angular velocity ω. This is known as the *primary* force.

The second term may be rewritten as $(W/g).(2\omega)^2.(r^2/4l).\cos 2\theta$ in which form it is evident that it represents the projection, on to the line of stroke, of the inertia force due to a weight W concentrated at an imaginary crank, of radius $r^2/4l$, and rotating at an angular velocity 2ω, in the same plane as the main crank. This is known as the *secondary* force.

For complete primary balance of a set of reciprocating weights,

(i) The primary force polygon must close, i.e. $\Sigma Wr = 0$, and

(ii) The primary couple polygon must close, i.e., $\Sigma Wrx = 0$.

Taking the plane of 4, that of the required weight, as a reference plane and drawing the couple polygon (moments about 4) so that it closes, we get for this polygon a triangle as in Fig. 104 (*a*). Note that since all radii are equal we need only consider Wx in each case. The directions of the sides of this triangle give the relative angular position of the cranks 1, 2, and 3. The crank angle 4 and the necessary weight are found by drawing the force polygon as in Fig. 104 (*b*), the vectors being drawn radially from the centre of the crankshaft.

The closing line scales 1·74 ton and this is the required weight. The relative crank angles are shown in the figure.

24. In a four-crank engine the two middle cranks, 2 and 3, are set at an angle of 105° with each other, and their reciprocating masses are respectively 2 520 lb. and 2 240 lb. Find the reciprocating mass and the relative crank angle of each of the two end cranks, 1 and 4, in order that all reciprocating masses may be balanced for primary forces and couples. The distances between the crank centres, measured along the centre line of the crankshaft, are 37½ in. between 1 and 2; 60 in. between 2 and 3; and 45 in. between 3 and 4. (*I.M.E.*)

Ans. (1) 1 820 lb.; (2) 1 499 lb.
97° between 3 and 1, and 58° between 1 and 4.

25. The dimensions of a coupled, two-cylinder locomotive with cranks at right angles are: distance between wheels, 5 ft.; between cylinders, 7 ft.; and between coupling-rod centres, 8 ft. The weight of the reciprocating parts is 510 lb., and of the rotating parts 200 lb., per cylinder. The weight of each coupling-rod is 500 lb. The coupling-rod cranks are in the same directions as the corresponding main cranks and all crank radii are 1 ft. Determine the weight and angular position of the weights to be placed on the driving wheels and the coupled wheels at an effective radius of 2½ ft. in order to balance all the rotating and two-thirds of the reciprocating weights.

State in what way any forces or couples which still remain unbalanced tend to act. (*L.U.*)

. SOLUTION (Fig. 105). The weight to be balanced per cylinder, assumed concentrated at the crank-pin,

$$= 200 + (2/3)510 = 540 \text{ lb. at crank-pin.}$$
$$= 500/2 = 250 \text{ lb. at coupling-rod crank-pin.}$$

Driving wheels. We proceed to find, analytically, the weights W_1 and W_2 which must be placed in the plane of

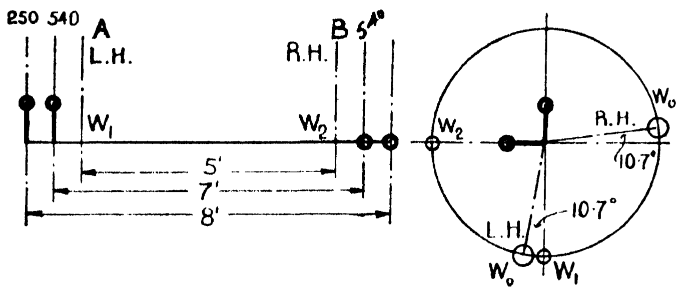

FIG. 105

the wheels *A* and *B*, opposite the left-hand and right-hand cranks respectively to balance the cranks.

Taking moments about *B* and considering the left-hand crank,

$$W_1 \times 2{\cdot}5 \times 5 = (540 \times 1 \times 6) + (250 \times 1 \times 6{\cdot}5)$$
$$\therefore \qquad W_1 = 389 \text{ lb.}$$

Taking moments about *A* and remembering that W_2 is on the opposite side of the reference plane through *A*,

$$W_2 \times 2{\cdot}5 \times 5 = (540 \times 1 \times 1) + (250 \times 1 \times 1{\cdot}5)$$
$$\therefore \qquad W_2 = 73{\cdot}2 \text{ lb.}$$

This is in line with the right-hand crank.

$$\tan \phi = W_2/W_1 = 73{\cdot}2/389 = 0{\cdot}1881 = 10{\cdot}65^\circ$$
$$W_0 = \sqrt{[(389)^2 + (73{\cdot}2)^2]} = 396 \text{ lb.}$$

The position of W_0 in each wheel is shown in Fig. 105.

A similar method may be adopted to find the balance weights for the coupled wheels and this, and the remainder of the problem, are left to the student.

26. In a three-cylinder, three-crank engine, the reciprocating parts for each cylinder weigh 1 800 lb. The stroke is 2 ft., length of connecting-rod 4 ft. 6 in., distance between cylinder centre lines 3 ft., and the cranks are set at 120°. Estimate the out-of-balance primary and secondary couples when the engine is running at 180 r.p.m. *(I.M.E.)*

SOLUTION. The crank radius is 1 ft. and we have $\omega = 2\pi \times 180/60 = 6\pi$ radn./sec.

Out-of-balance primary couple. Taking the plane of 1 (Fig. 106) as a reference plane we have,

$$W_2x_2 = 1\,800 \times 3 = 5\,400 \text{ lb. ft.}$$
$$W_3x_3 = 1\,800 \times 6 = 10\,800 \text{ lb. ft.}$$

The couple polygon is drawn as in Fig. 106 (*b*) and the closing line scales 9 369.

$\therefore$ Out-of-balance primary couple

$$= \Sigma Wrx(\omega^2/g)$$
$$= 9\,369 \times 1 \times 36\pi^2/32{\cdot}2 = 103\,300 \text{ lb. ft.}$$

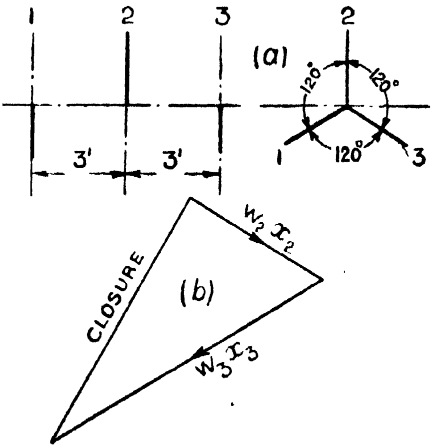

FIG. 106

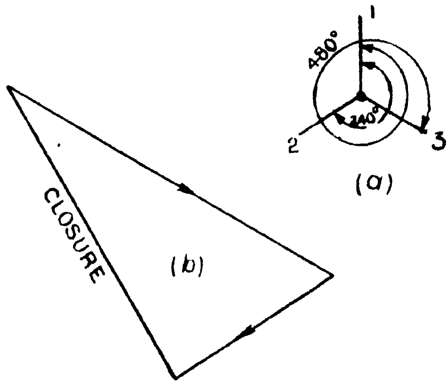

FIG. 107

Out-of-balance secondary couple. The crank angles for the imaginary secondary cranks are shown in Fig. 107 (*a*). The radii of these imaginary cranks are all $r^2/4l = 1/18$ ft. Draw the secondary couple polygon, assuming unit radius

for each crank as in Fig. 107 (*b*). This gives a closing line of 9 369, as before.

$\therefore$ Out-of-balance secondary couple

$$= (9\,369/32{\cdot}2) \times (2\omega)^2 \times (r^2/4l)$$

$$= \frac{9\,369 \times 144\pi^2 \times 1}{32{\cdot}2 \times 18} = 22\,960 \text{ lb. ft.}$$

27. The diagram (Fig. 108) shows the arrangement of a four-cylinder symmetrical engine. Prove that this engine

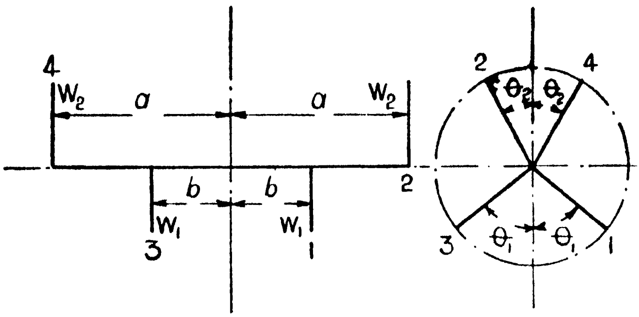

FIG. 108

may be balanced for primary forces and couples and for secondary forces, and find the necessary relations which must exist between the crank angles and pitches.

28. In the Michell swash-plate engine shown diagrammatically in Fig. 109, the swash-plate may be regarded as having been cut from a steel cylinder with axis XX' by two parallel planes inclined to this axis by an angle α. The shaft of the engine to which the plate is fixed rotates in bearings about the axis XX'. There are n fixed cylinders on each side of the plate and they are distributed uniformly round the axis. In the diagram one pair of cylinders only has been shown with the pistons, rods, and slippers which transmit the driving forces. The plane of the figure is perpendicular to the plane of the swash plate.

Show that the inertia forces acting on all the moving parts of the engine can be combined into a single couple in a plane rotating with the engine and containing the axes AA'

and XX', and that its moment can be expressed in the form $(\frac{1}{2}I - nMr^2)\omega^2 . \cot \alpha$,

where I = moment of inertia of swash-plate about XX',

M = total mass of one piston, rod, slipper, etc.,

r = radial distance between XX' and the axis of any cylinder,

ω = angular velocity of engine.

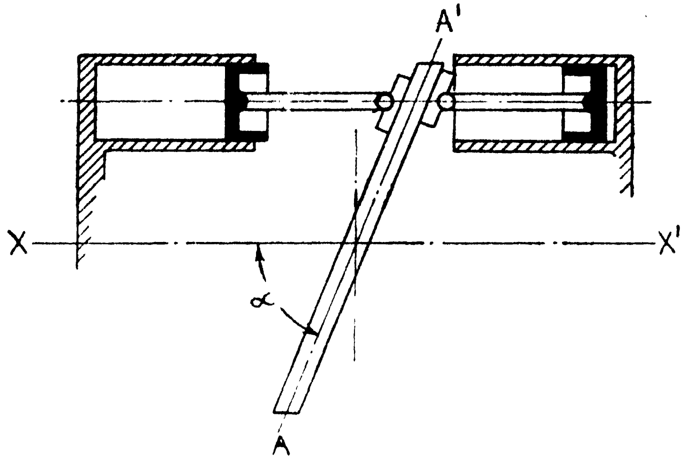

FIG. 109

Sketch a simple arrangement of balance weights by means of which the engine could be perfectly balanced. (*W.S.S.*)

29. Describe carefully how a connecting-rod may be replaced (for purposes of balancing) by two point masses, one at the crank-pin, and one elsewhere along the rod.

30. Show in isometric projection the arrangement of cranks in an eight-cylinder engine which is designed so that the dynamic effects are balanced for both primary and secondary actions. Draw the force and couple polygons for primary and secondary effects.

10. GYROSTATICS AND STEERING GEARS

Note on Gyrostatic Action. If a rigid body be rotating about an axis, and a couple is applied which tends to turn the body about an axis perpendicular to the axis of spin, then the body will rotate, or tend to rotate, about a third axis which is perpendicular to both of the other axes. This is known as a *gyrostatic action* and the rotation about the third axis is called *precession*.

It may be shown that if

ω = angular velocity of spin,

I = moment of inertia about axis of spin,

ω' = angular velocity of precession,

then the couple applied is given by

$$C = I\omega\omega' = \text{rate of change of angular momentum.}$$

√1. Two fly-wheels *W* together with their axle have a weight of 60 lb. and a radius of gyration of 2 ft. They spin

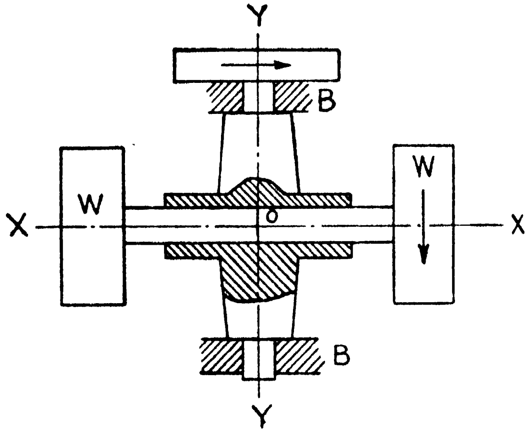

Fig. 110

freely about *XX* at 2 000 r.p.m. The bearing which carries the shaft rotates in fixed bearings, *BB*, about *YY* at 1 000 r.p.m. What is the unbalanced couple on the frame *BB*? Neglect friction.

Solution (Fig. 110). The couple is given by

$C =$ Rate of change of angular momentum.

$C = I \, . \, \omega \, . \, \omega'$

$$I\omega = \frac{60}{32{\cdot}2}(2)^2 \times \left(\frac{2\pi \times 2\ 000}{60}\right) = 1\ 560 \text{ lb. ft. sec. units}$$

$$\omega' = 2\pi \times 1\ 000/60 = 104{\cdot}66 \text{ radn./sec.}$$

$$\therefore \quad C = (1\ 560 \times 104{\cdot}66) = 163\ 100 \text{ lb. ft.}$$

This acts about an axis through O normal to plane XOY, and in the figure the sense of C is clockwise for the rotations indicated.

2. A fly-wheel of weight 2 tons and radius of gyration 3 ft. is keyed to its shaft so that its plane makes an angle of 2° with a plane normal to the shaft. If the shaft rotates at 250 r.p.m., find the unbalanced couple on the bearings due to the obliquity of the fly-wheel.

Solution (Fig. 111). Considering the rate of change of angular momentum, when the shaft has turned through an

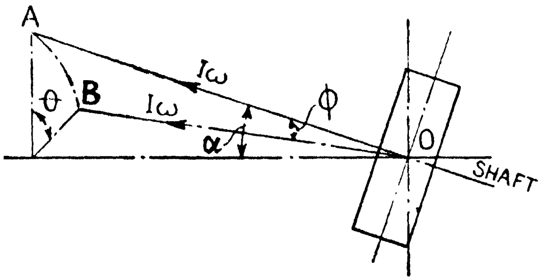

Fig. 111

angle θ the vector OA representing angular momentum has turned through an angle ϕ such that

$$I\omega \, . \, d\phi \, . = AB = I\omega \, . \sin\alpha \, . \, d\theta.$$

$$\therefore \qquad d\phi = \sin\alpha \, . \, d\theta.$$

Hence the angular velocity of precession is,

$$\omega' = d\phi/dt = \sin\alpha \, . \, (d\theta/dt) = \omega \, . \sin\alpha.$$

The gyrostatic couple acting on the bearings is therefore given by,

$$C = I\omega\omega' = I\omega^2 . \sin\alpha$$

$$= \frac{2 \times 2\,240}{32{\cdot}2}(3)^2 \times \left(\frac{2\pi \times 250}{60}\right)^2 \times \sin 2^\circ.$$

$$= 29\,920 \text{ lb. ft.}$$

This couple acts in the plane containing the shaft and the axis of the fly-wheel, and rotates with this plane.

3. A ventilating fan together with the motor armature has a weight of 5 lb. and a radius of gyration of 2 in. It rotates at 5 000 r.p.m. and is mounted on a vertical spindle perpendicular to its axis by a universal joint so that the c.g. overhangs the centre of the spindle by $\frac{1}{8}$ in. Neglecting all frictional effects find the speed at which the fan and motor rotate about the vertical axis. *Ans.* 0·44 r.p.m.

4. A pair of driving wheels and shaft of a locomotive have a weight of 2 tons and a radius of gyration of 3 ft. The outer diameter of the wheels is 6 ft. 6 in., and the distance between centres of bearings is 4 ft. If the locomotive is rounding a curve of radius 500 ft. at 60 m.p.h., find the additional reactions on the bearings due to gyrostatic action.

(*Hint.* Take radius of track to be mean radius and neglect slipping.)

Ans. Reactions = 1 492 lb.

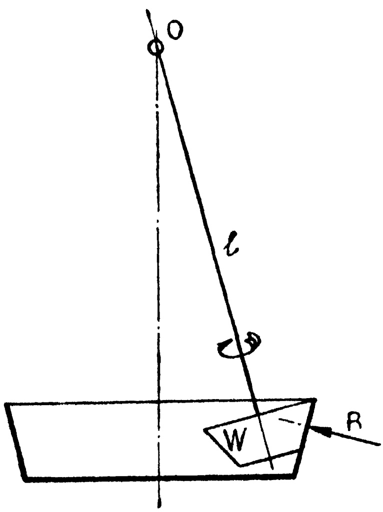

FIG. 112

5. A grinding mill is shown in Fig. 112. The wheel *W* is rigidly attached to the shaft which is rotated at constant speed ω radn./sec. through a universal coupling at *O*. Initially the shaft *l* hangs vertically. Explain how it may be set in motion as a mill and obtain an expression for the normal pressure R between the mill and the mortar.

6. A steam turbine, mounted in a boat, makes 3 000 r.p.m. The effective moment of inertia of the rotor, shaft, and propeller is 0·5 ft. ton units. Find the magnitude and direction of the couple acting on the hull when the boat describes a circular path, making a complete turn in 12 sec. Prove any formula you use. (*L.U.*)

SOLUTION. The proof of the formula—

$$\text{Couple} = I\omega\omega' \text{ is left to the student.}$$

We have $\omega = 2\pi \times 3\,000/60 = 314$ radn./sec.

Since the boat makes one complete turn in 12 sec.

$$\omega' = 2\pi/12 = \pi/6 \text{ radn./sec.}$$

Hence the couple acting on the hull is

$$C = I\omega\omega' = 0{\cdot}5 \times 314 \times \pi/32{\cdot}2 \times 6$$
$$= 2{\cdot}56 \text{ ton ft.}$$

This acts about an axis perpendicular to the direction of motion.

7. A locomotive weighs 80 tons and travels at 50 m.p.h. on a curve of 2 000 ft. radius. The height of the centre of gravity above rail level is 8 ft. The outer rail of the curve is $4\frac{1}{2}$ in. higher than the inner and the rails are 5 ft. from centre to centre. Assuming that the wheels have a radius of gyration of 2 ft. and a total weight of 7 tons, find the overturning moment of the locomotive due to gyrostatic action of the wheels and due to the outer rail being lower than is desirable for the given speed. (*I.M.E.*)

Ans. 6·47 ton ft.

8. A motor cycle and rider together weight 350 lb., their centre of mass being 18 in. above the ground when the cycle is vertical and 24 in. in front of the axis of the back wheel.

The wheels of the cycle each weigh 20 lb.; their outer diameter is 26 in., and their radius of gyration 12 in. The wheel base is 54 in. The engine has a fly-wheel weighing 48 lb., with a radius of gyration of 7 in.; the axis of the crankshaft is horizontal and in the plane of the road-wheels, the direction of rotation being clockwise when viewed from the rear. At 30 m.p.h. the engine makes 2 400 r.p.m.

Find (*a*) the inclination to the vertical, and (*b*) the reactions between the road and each wheel when rounding a curve to the right of 120 ft. radius at 30 m.p.h. (*L.U.A.*)

Ans. (*a*) 28·3°
(*b*) 178·5 lb. front.
171·4 lb. rear.

9. An aeroplane is propelled by a rotary engine which rotates anti-clockwise from the front, at a speed of 1 000 r.p.m. The moment of inertia of the complete engine rotating parts and propeller is 10 lb./ft.^{2} If the aeroplane makes a half turn to the right in 12 sec., find the gyroscopic couple acting on it and its effect.

Ans. 4·25 lb. ft. tending to raise the tail.

10. The crankshaft and fly-wheel of a motor car have a moment of inertia of 150 lb. ft.^{2} and rotate at 1 000 r.p.m. The shaft is parallel to the direction of motion, and rotates clockwise as viewed from the rear. If the car rounds a curve 25 ft. radius at 15 m.p.h., what is the gyrostatic couple on the frame? If the turn is (*a*) to the left, (*b*) to the right, state the effect of the couple on the car.

Ans. 429 lb. ft.
(*a*) Back springs depressed.
(*b*) Front springs depressed.

11. Discuss the motion of a top and show how the angle of inclination to the vertical and the speed of precession may be found.

12. A motor cycle and rider together weigh 300 lb. The fly-wheel runs in the same sense as the road wheels at 2 500 r.p.m. If the rider turns a corner of 25 ft. radius at 30 m.p.h., find approximately the distance which he must move the centre of gravity of himself and the machine in order that the fly-wheel may precess. Take the moment of inertia of the fly-wheel to be 3 lb. ft.^{2}

Ans. 1·72 in.

13. The wheel-base of a motor car is of length *H*. The front axle has a length *b* and the steering wheels pivot about the ends of this axle.

State the configuration of these wheels for correct steering and thus obtain a simple formula connecting the angles between the planes of the steering wheels and the front axle, the wheel base, and the length of the axle, which must hold for all correct steering positions.

14. The mechanism shown in Fig. 113 is the Davis steering gear. Show that it gives correct steering in all positions

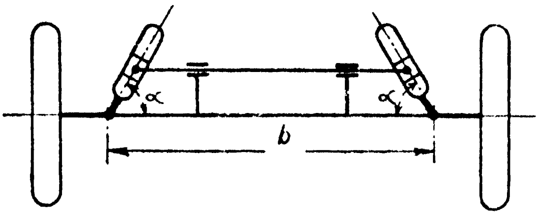

FIG. 113

if $\cot \alpha = b/2H$ where H is the length of the wheel base. Why is this gear not practicable?

15. The Ackerman steering gear makes use of the four-bar mechanism as shown in Fig. 114. If the length AB of the

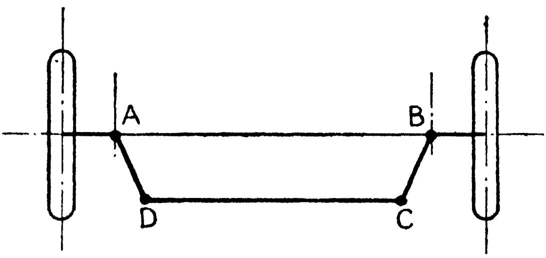

FIG. 114

axle is 4 ft. and the wheel base of the car is 7 ft., design the gear so that the steering will be correct when turning in a circle 15 ft. radius.

SOLUTION. From Fig. 115 $\tan \theta = 7/13 \quad \therefore \theta = 28{\cdot}3^\circ$ and $\tan \phi = 7/17 \quad \therefore \phi = 22{\cdot}4^\circ$.

In this position of the gear the inclination of DC to AB is only a few degrees and consequently the projections of CD and C_1D_1 on AB may be assumed equal. Then we must have

Projection of *arc* D_1D on AB = Projection of *arc* C_1C on AB.

Hence if $\alpha = \angle D_1AB = \angle C_1BA$ which is required and $AD_1 = a$, we have,

$$a[\cos(\alpha - \theta) - \cos\alpha] = a[\cos\alpha + \cos(\phi + \alpha)]$$

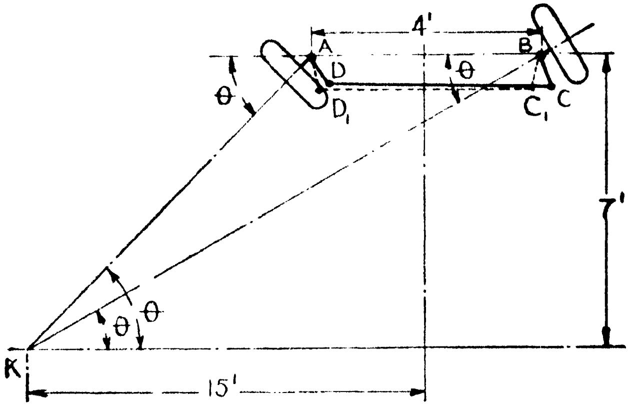

FIG. 115

or $\quad \cos\alpha \,.\, \cos\theta + \sin\alpha \,.\, \sin\theta - \cos\alpha$

$$= \cos\alpha + \cos\phi \,.\, \cos\alpha - \sin\phi \,.\, \sin\alpha$$

$$\cos\alpha\,(\cos\theta - \cos\phi - 2)$$

$$= -\sin\alpha\,(\sin\theta + \sin\phi)$$

$\therefore \quad \tan\alpha = (2 + \cos\phi - \cos\theta)/(\sin\theta + \sin\phi).$

Substituting, $\tan\alpha = 2{\cdot}044/0{\cdot}8552 = 2{\cdot}39$

$\therefore \quad \alpha = 67{\cdot}3°.$

This is the important dimension of the gear.

11. VIBRATIONS OF ELASTIC BODIES—WHIRLING OF SHAFTS

Note on the Energy Method of Obtaining the Equations of Motion. This method of dealing with vibration problems makes use of the relation,

Total energy of system = constant.

In order to form the energy equation in a particular case some assumption must be made regarding the deflected form of the system during vibration. Lord Rayleigh has shown that any assumed shape not strikingly unlike the actual deflected shape leads to results which are very nearly accurate. The most usual assumption is that the deflected shape during vibration has the same form as the statical deflected shape.

1. A helical spring of stiffness k and axial length l is rigidly fastened at its upper end, and supports a weight W at its lower end. Find the period of a small vertical vibration taking account of the inertia of the spring.

SOLUTION. The assumption made in this case is that the vertical displacement of a point P on the spring due to a displacement x of the free end is xz/l. The total energy of the system at any instant is made up of—

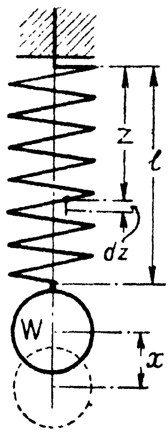

FIG. 116

(i) Kinetic energy of system.
(ii) Potential energy of system.

The potential energy consists of—

(*a*) Potential energy due to position of weight W.
(*b*) Potential energy due to deformation of spring.

If the system starts from rest we must have after time t,

Gain of energy = loss of energy.

If x is the displacement of W at time t,

$$\text{K.e. of weight } W = (W/2g)(dx/dt)^2$$

$$\text{K.e. of spring} = \int_0^l \frac{w}{2g} \cdot dz\left(\frac{z}{l} \cdot \frac{dx}{dt}\right)^2 = \frac{w}{2g} \cdot \frac{l}{3} \cdot \left(\frac{dx}{dt}\right)^2$$

where w = weight of unit axial length of spring.

$$\text{Strain energy gained by spring} = Wx + \tfrac{1}{2}kx^2.$$

$$\text{Potential energy lost by weight } W = Wx.$$

Equating gain of energy to loss of energy we get,

$$\frac{W}{2g}\left(\frac{dx}{dt}\right)^2 + \frac{w}{2g} \cdot \frac{l}{3} \cdot \left(\frac{dx}{dt}\right)^2 + Wx + \frac{kx^2}{2} = Wx$$

$$\therefore \qquad \frac{1}{2g}\left(W + \frac{wl}{3}\right) \cdot \left(\frac{dx}{dt}\right)^2 + \frac{kx^2}{2} = 0.$$

Differentiating this equation with respect to time, and dividing by dx/dt,

$$\frac{W + wl/3}{g} \cdot \frac{d^2x}{dt^2} + kx = 0.$$

This is the equation of motion of the system and obviously represents a simple harmonic vibration of period,

$$t = 2\pi\sqrt{[(W + wl/3)/kg]} \text{ sec.}$$

This result should be compared with that of Problem 3, Section 4, when it will be seen that the period above is the same as that for a spring of negligible mass with a weight (W + $\frac{1}{3}$ weight of spring) at its free end.

2. A weight W is vibrating in a vertical plane at the end of a uniform horizontal cantilever of length l. Show that the period of a small vibration is the same as that for a weightless cantilever of the same dimensions with a load $(W + 33\, wl/140)$ at its free end, where w = weight of unit length of the cantilever.

(*Hint.* Assume the deflected form during vibration is the same as that produced by a static load at the free end.)

3. A horizontal cantilever of length 4 ft. and uniform cross-section 3 in. wide by $\frac{1}{4}$ in. deep performs small vibrations in a vertical plane. Given that the material of the cantilever weighs 480 lb./ft.^{3} and $E = 30 \times 10^6$ lb. per in.^{2}, find the approximate frequency of natural vibration.

(*Hint.* Proceed as in Problem 2 and finally substitute $W = 0$.) *Ans.* $f = 3{\cdot}63$ per sec.

4. A uniform beam of length l is simply supported at its ends. A weight W is concentrated at its mid-point. If w is the weight of unit length of the beam show that the frequency of small natural vibrations is given by

$$f = \frac{1}{2\pi}\sqrt{\left[\frac{48\mathrm{EI}g}{(\mathrm{W} + 17wl/35)l^3}\right]}$$

where E is Young's modulus for the material of the beam.

5. A load of 2 000 lb. is supported at mid-span by a simply supported horizontal beam of span 24 ft., of I-section 10 in. deep, having a second moment of 211·5 in.^{4} about the neutral axis. Neglecting viscous resistances and the inertia of the beam itself, calculate the natural period of vibration of the system.

An alternating load due to a mass of 12 lb. at a radius of 9 in. revolving at 360 r.p.m. is superimposed on the given load. Calculate the amplitude of the forced vibration and the additional maximum stress in the beam due to this. E for beam $= 30 \times 10^6$ lb./in.^{2} (*L.U.A.*)

Ans. 0·1266 sec.
2·907 in.
26·65 ton/in.^{2}

6. Using the Rayleigh method, show that the period of torsional vibration of a circular shaft of length l, fixed at one end and carrying a disc at the free end, is the same as that for a shaft of negligible mass, of the same dimensions, having at its free end a disc of moment of inertia

$$(\mathrm{I} + il/3),$$

where I = moment of inertia of disc and

i = moment of inertia of shaft per unit length.

Hence, calculate the period for a shaft of length 6 ft., 6 in. diameter and carrying at the end a disc 4 ft. diameter, 4 in. thick. The material of the shaft weighs 450 lb./ft.^{3} $C = 12 \times 10^6$ lb./in.^{2}

Ans. 0·177 sec.

7. A beam 15 ft. long and freely supported at the ends, carries masses of 5, 2, and 4 tons respectively at the points 3 ft., 7·5 ft., and 12 ft. from one end. The mass of the beam itself may be neglected.

It is found that the deflection of the beam is 0·25 in. when the central load only is applied. Deduce the natural frequency of vibration of the loaded beam, stating carefully what assumptions you have made. If it were considered necessary to take the mass of the beam itself into account, indicate briefly how this might be done.

Note. The deflection y of a freely supported beam of span l, at a point distant x from one end, due to a load W at a distance a from that end ($a \geqq x$) is given by,

$$y = \frac{Wl^3}{6EI} \cdot \frac{x}{l} \cdot \frac{l-a}{l} \cdot \left[1 - \left(\frac{x}{l}\right)^2 - \left(\frac{l-a}{l}\right)^2\right].$$

(*L.U.A.*)

Ans. 3·93 per sec.

(*Hint.* Use the energy method, calculating statical deflections by given formula, having first found 1/EI from the given deflection.)

8. The shaft shown in the diagram (Fig. 117) carries two

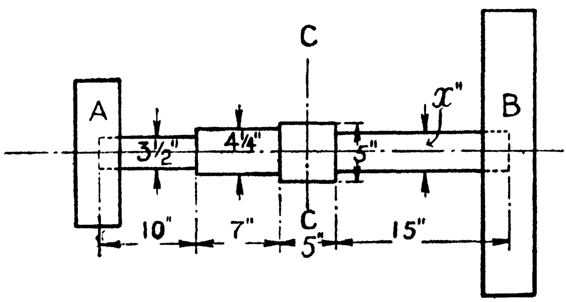

FIG. 117

heavy masses at A and B and is driven by a light gear situated at CC. The weight of A is 800 lb. and its radius of

gyration is 27 in.; the corresponding values for B are 1 200 lb. and 33 in.

The shaft diameter between CC and B, marked x'', is undecided. Assuming it to be $3\frac{1}{2}$ in., determine the frequency of free torsional oscillations of the system. Thereafter, determine what x should be if the node of the vibration is to be in the plane CC of the drive. Deduce any formula used. Rigidity modulus $= 12 \times 10^6$ lb./in.^{2}

SOLUTION. The frequency of free torsional vibration of such a system is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{\pi C d^4(I_A + I_B)}{32l \,.\, I_A I_B}}$$

where l is the length of shaft between A and B which corresponds to a constant diameter d. The proof of this formula is left to the student.

To apply this we must find the length l equivalent to a constant diameter d, say, equal to $3\frac{1}{2}$ in. Note that the angle of twist of such a shaft is proportional to the length and inversely proportional to the fourth power of the diameter. Hence, without altering the angle of twist, a shaft of length l_1 and diameter b_1 may be replaced by one of length l_2, and diameter b_2 by satisfying the condition,

$$l_1/l_2 = (b_1/b_2)^4.$$

Reducing the shaft to an equivalent length l of diameter $d = 3\frac{1}{2}$ in. we have,

(i) the $4\frac{1}{4}$ in. section—

$$7/l_2 = (4{\cdot}25/3{\cdot}5)^4 \qquad \therefore\ l_2 = 3{\cdot}22 \text{ in.}$$

(ii) the 5 in. section—

$$5/l_2 = (5/3{\cdot}5)^4 \qquad \therefore\ l_2 = 1{\cdot}2 \text{ in.}$$

Hence the equivalent length of shaft is,

$$l = (10 + 15 + 3{\cdot}22 + 1{\cdot}2) = 29{\cdot}42 \text{ in.}$$

$$I_A = \frac{800}{32{\cdot}2}\cdot\left(\frac{27}{12}\right)^2 = 125{\cdot}8 \text{ units}$$

$$I_B = \frac{1\,200}{32{\cdot}2}\cdot\left(\frac{33}{12}\right)^2 = 281{\cdot}8 \text{ units.}$$

Substituting

$$= f\frac{1}{2\pi}\sqrt{\left[\frac{\pi \times 12 \times 10^6 \times (3\cdot5)^4(125\cdot8 + 281\cdot8) \times 12 \times 144}{(12)^4 \times 32 \times 29\cdot42 \times 125\cdot8 \times 281\cdot8}\right]}$$

$= 12\cdot09$ vibrations per second.

The position of the node is determined by the relation,

$$I_A/I_B = BC/AC \text{ and}$$

$$AC = 13\cdot82 \text{ in. of equivalent shaft.}$$

Hence $$BC = \frac{13\cdot82 \times 125\cdot8}{281\cdot8}$$

$= 6\cdot17$ in. of equivalent shaft.

The actual length of the x in. diameter section is 15 in., and its equivalent shaft length is $(6\cdot17 - 0\cdot6) = 5\cdot57$ in. To find x we have,

$$15/5\cdot57 = (x/3\cdot5)^4 \text{ whence } x = 4\cdot48 \text{ in.}$$

9. The propeller and engine fly-wheel of a ship have moments of inertia of 70 and 90 ton ft.^{2} respectively, and are mounted 40 ft. apart on a hollow shaft 15 in. outside and 12 in. inside diameter. Calculate the frequency of torsional oscillations of the shaft.

In the above arrangement the fly-wheel is between the propeller and the engine, which is 58 ft. from the propeller. In order to lessen vibration, the size of the fly-wheel is reduced and a second fly-wheel is mounted on the shaft, 68 ft. from the propeller. By this means one of the two nodes of the oscillations due to the three masses occurs at the engine itself and the frequency of the oscillations is 190 per min.

Calculate the moments of inertia of the fly-wheels. Neglect the mass of the shaft and take C for the shaft as 12×10^6 lb./in.^{2} (*L.U.A.*)

Ans. $f = 7\cdot65$ per sec.
$I_2 = 7\cdot3$ ton ft.^{2} $I_3 = 8\cdot89$ ton ft.^{2}

10. Define *whirling speed* as applied to a rotating shaft and show that for a long unloaded uniform shaft, running in

short bearings, the lowest speed at which whirling can occur is given by,

$$\omega = (\pi^2/l^2)\sqrt{(gEI/w)}$$

where w is the weight per unit length of the shaft.

11. A light shaft running in short bearings carries a load the dimensions of which are small compared with the length of the shaft. Find the lowest speed at which whirling will occur.

SOLUTION. Treating the shaft as a light beam carrying a concentrated load W (Fig. 118), we have

$$\text{Deflection under load W} = y = Wa^2b^2/3EIl$$

where a and b are the lengths of the parts into which W divides the length l.

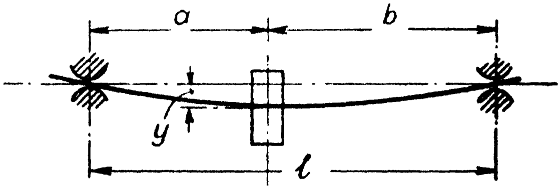

FIG. 118

Since the shaft is rotating, the load W is replaced by the centrifugal force $(W/g) \cdot \omega^2 \cdot y$, and in the critical state, when the centrifugal force is just balanced by the restoring force due to the stiffness of the shaft, we have

$$y = (W/g)(\omega^2 ya^2b^2/3EIl)$$

or $$\omega_{critical} = \sqrt{(3EIgl/Wa^2b^2)}$$

This is the required whirling speed.

12. A shaft carries a wheel at mid-span between its two bearings, and measurements made in a static balancing machine show that the centre of gravity is displaced 0·10 in. from the axis, although the shaft is true.

When the shaft is running at 600 r.p.m. it is found to be deflected 0·03 in. by the unbalanced centrifugal force. Estimate the critical speed at which the shaft may be expected to whirl. (*I.M.E.*)

SOLUTION. Let h (Fig. 119) denote the distance of the centre of gravity from the axis of the shaft, and y the deflection of the point C from the axis of rotation at speed ω radn./sec., then, due to rotation, a force F acts radially at G, given by,

$$F = (W/g)\omega^2(y + h).$$

Now it is known that if a force F acts at the mid-point of a uniform shaft, freely supported, it will cause a deflection,

$$y = Fl^3/48EI, \text{ which gives}$$
$$F = 48EIy/l^3.$$

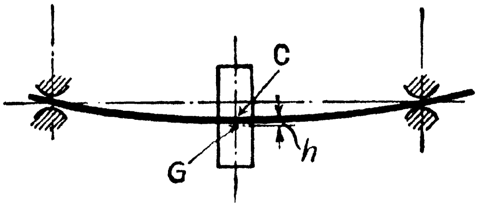

FIG. 119

Equating these two expressions for F we get,

$$48EIy/l^3 = (W/g)\omega^2(y + h)$$

$$\therefore \quad y = \frac{(W/g)\omega^2 h}{(48EI/l^3 - \omega^2 W/g)} = \frac{h}{(48EIg/\omega^2 Wl^3) - 1}$$

y becomes infinite when

$$\omega^2 = 48EIg/Wl^3$$

and this value gives the whirling speed.

Also we have $\quad y = 0{\cdot}03$ in. at 600 r.p.m.

and $\quad h = 0{\cdot}10$ in.

$$\therefore \quad 0{\cdot}03 = \frac{0{\cdot}10}{\dfrac{48EIg}{Wl^3[(2\pi \times 600)/60]^2} - 1}$$

giving $\quad 48EIg/Wl^3 = \omega^2_{critical} = 400 \times \pi^2 \times 13/3$

$$\therefore \quad \omega_{critical} = 130{\cdot}7 \text{ radn./sec.} = 1\,248 \text{ r.p.m.}$$

13. A shaft designed to carry a rotor that weighs 5 tons is found to be deflected 0·01 in. when a test load of 2 tons is

suspended on it by slings from the seating prepared for the rotor. Estimate approximately the critical whirling speed that may be expected when the rotor is fitted. (*I.M.E.*)

Ans. 343 r.p.m.

14. An unloaded uniform shaft of length l and weight w lb. per unit length runs in "long" bearings. Show that the critical speed at which whirling occurs is given by

$$\cos ml \,.\, \cosh ml \,. = 1$$

where $$m^4 = w\omega^2/g\text{EI}.$$

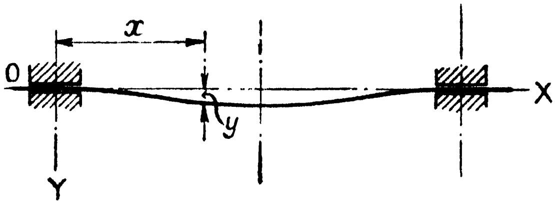

FIG. 120

SOLUTION. Measuring x (Fig. 120) along the axis of rotation and taking y as the deflection at x.

The centrifugal force per unit length of shaft

$$= (w/g) \,.\, \omega^2 \,.\, y.$$

It is shown in textbooks on Strength of Materials that

$$\text{EI} \,.\, (d^4y/dx^4) = \text{load per unit length}$$

so that, in this case,

$$\text{EI} \,.\, (d^4y/dx^4) \,. = w/g \,.\, \omega^2 \,.\, y$$

or $$d^4y/dx^4 - m^4y = 0 \text{ where } m^4 = w\omega^2/g\text{EI}.$$

This is the equation of the strained central line of the shaft.

The general solution of this equation is,

$$y = A \sin . mx + B \sinh mx + C \cos mx + D \cosh mx.$$

Since the shaft runs in "long" bearings the conditions approximate to fixed ends and so $y = 0$ and $dy/dx = 0$ at

both ends, and since we have taken the origin at one end we get

$$C = -D \text{ and } A = -B \text{ and}$$

$$C = -A\,(\sin ml - \sinh ml)/(\cos ml - \cosh ml)$$

and the equation becomes,

$$y = A\left[(\sin mx - \sinh mx) - \left(\frac{\sin ml - \sinh ml}{\cos ml - \cosh ml}\right)(\cos mx - \cosh mx)\right]$$

Finally the condition $dy/dx = 0$ when $x = l$ requires that either $A = 0$, when there is no deflection, or

$$(\cos ml - \cosh ml) + \left(\frac{\sin ml - \sinh ml}{\cos ml - \cosh ml}\right)(\sin ml + \sinh ml) = 0$$

This latter gives the condition of instability, and reduces to

$$\cos ml \,.\, \cosh ml = 1.$$

15. A steel shaft, 6 ft. long between bearings, has a diameter of 1 in. If the bearings are short, calculate the least whirling speed. $E = 30 \times 10^6$ lb./in.²

Ans. 920 r.p.m.

16. A steel shaft, 2 in. diameter, is carried at its ends in swivel bearings, 36 in. from centre to centre. A wheel of mass 400 lb. is carried with its mass centre 16 in. from one end.

Estimate the whirling speed, neglecting the mass of the shaft. Take E as 30×10^6 lb./in.² (*L.U.A.*)

Ans. 1 478 r.p.m.

17. Show that the frequency of transverse vibration of a loaded shaft is equal to the whirling speed of that shaft.

18. A uniform shaft is mounted at its ends in bearings which do not impose any flexural constraint upon it, and has keyed to it at the middle of its length a disc. Assuming the deflection form of the shaft to be that due to a central

load, prove that $\frac{17}{35}$ of the mass of the shaft should be added to that of the disc in estimating the whirling speed.

Find the whirling speed if the shaft is 1 in. diameter and 20 in. long, and the mass of the disc is 10 lb.

Density of steel 0·28 lb./in.^{3}, E for steel 30×10^6 lb./in.^{2} (*L.U.A.*)

SOLUTION. Since the whirling speed is equal to the frequency of transverse vibration of the shaft and disc, we simply proceed to find this latter using the energy method. It has already been shown in Problem 4 of this section that in estimating the frequency of transverse vibrations $\frac{17}{35}$ of the mass of the shaft should be added to the central load, and that the frequency is then given by

$$f = (1/2\pi)\sqrt{(g/\delta_{static})}$$

where

$$\delta_{static} = \text{static deflection under load.}$$

In this case

$$\delta_{static} = (1/48)(W + 17w/35)(l^3/EI)$$

where w = weight of shaft, and W = weight of disc.

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\left[\frac{32{\cdot}2 \times 48 \times 30 \times 10^6 \times 12 \times \pi \times 1}{(20\pi \times 0{\cdot}28 \times 17/140 + 10) \times 64 \times (20)^3}\right]}$$

$$= 50{\cdot}79 \text{ per sec.}$$

$\therefore$ Whirling speed

$$= 60 \times 50{\cdot}79 = 3\,047{\cdot}4 \text{ r.p.m.}$$

19. A uniform shaft of length l runs in a long bearing at one end and the other end is free. Show that the whirling speed is determined by

$$\cos ml \,.\, \cosh ml + 1 = 0$$

where $$m^4 = w\omega^2/g\mathrm{EI}.$$

Find the smallest root of this equation.

Ans. $ml = 1{\cdot}875.$

20. A heavy uniform shaft carries a concentrated load W at its mid-point, and rotates in "short" bearings. Making

use of the equation of the strained centre line of the shaft, show that the whirling speed is determined by,

$$\tan \tfrac{1}{2}ml - \tanh \tfrac{1}{2}ml = 4w/m\mathrm{W}$$

where $m^4 = w\omega^2/g\mathrm{EI}$ and w = weight per unit length of shaft.

(*Hint.* On account of symmetry $(dy/dx) = 0$ at $x = \tfrac{1}{2}l$, and shearing forces at centre on each side of W support the centrifugal force on W. Shearing force $= -\mathrm{EI} \, . \, (d^3y/dx^3\,.)$

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